Lattice QCD-Based 3D-Ising Equation of State

Universal Behaviors Near the Critical Point

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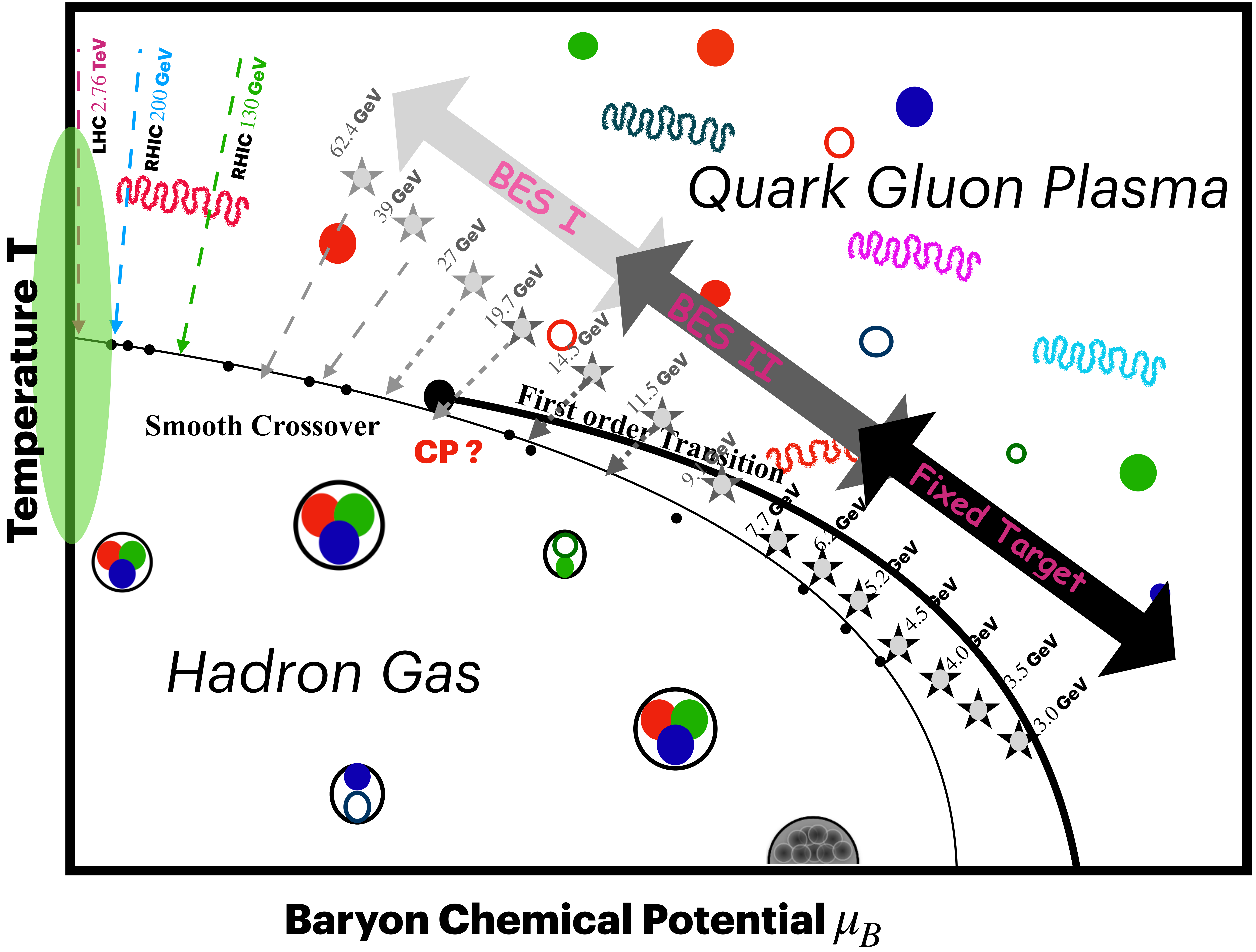
TEXAS SECTION AAPT PHYSICS EDUCATION

NuSTEAM Nuclear Science in Texas to Enhance and Advance Minorities

April, 3-5, 2025

QCD Phase Diagram



Experiments

- Finite density physics is achieved by lowering the $\sqrt{S_{NN}}$ in **BES program**
- Other experiments FAIR, NICA,J-PARC

Theoretical interpretation

- **Hydrodynamic models** simulate the evolution of the fireball produced in heavy-ion collisions
- The **Equation of State (EoS)** is essential in hydrodynamic models, as it governs the thermodynamic properties and evolution of the system

The EoS must capture relevant physics

Taylor: Lattice QCD results

Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!}\chi_{2n}(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^{2n}$$

[Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]

[Bazavov, A et al PhysRevD.95, 054504 (2017)]

$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\bigg|_{\mu_B=0}$$

[Borsanyi, S. et al JHEP 10 205 (2018)]

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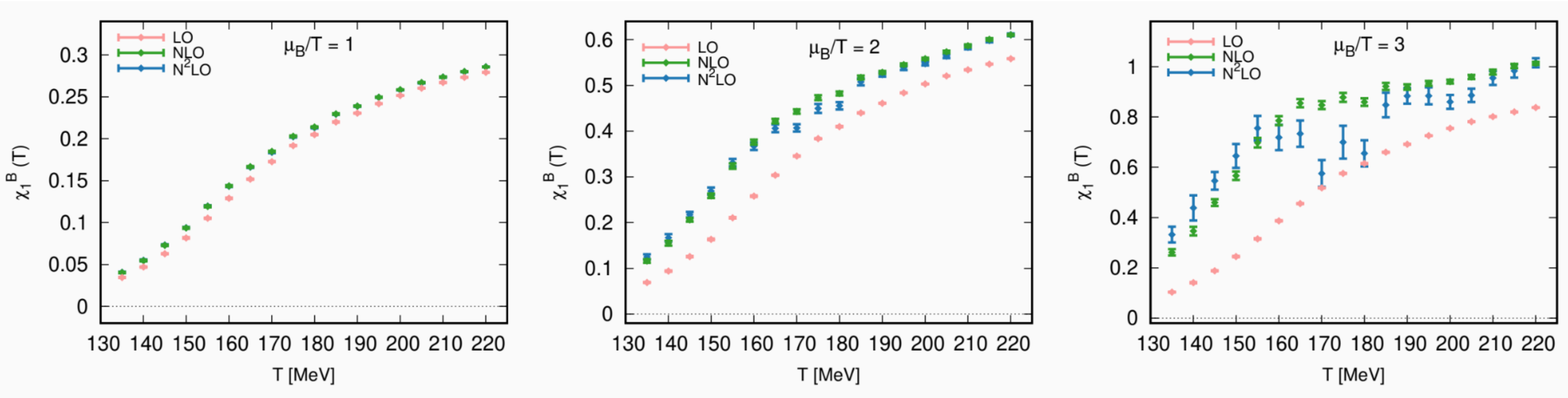
Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational effort**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]

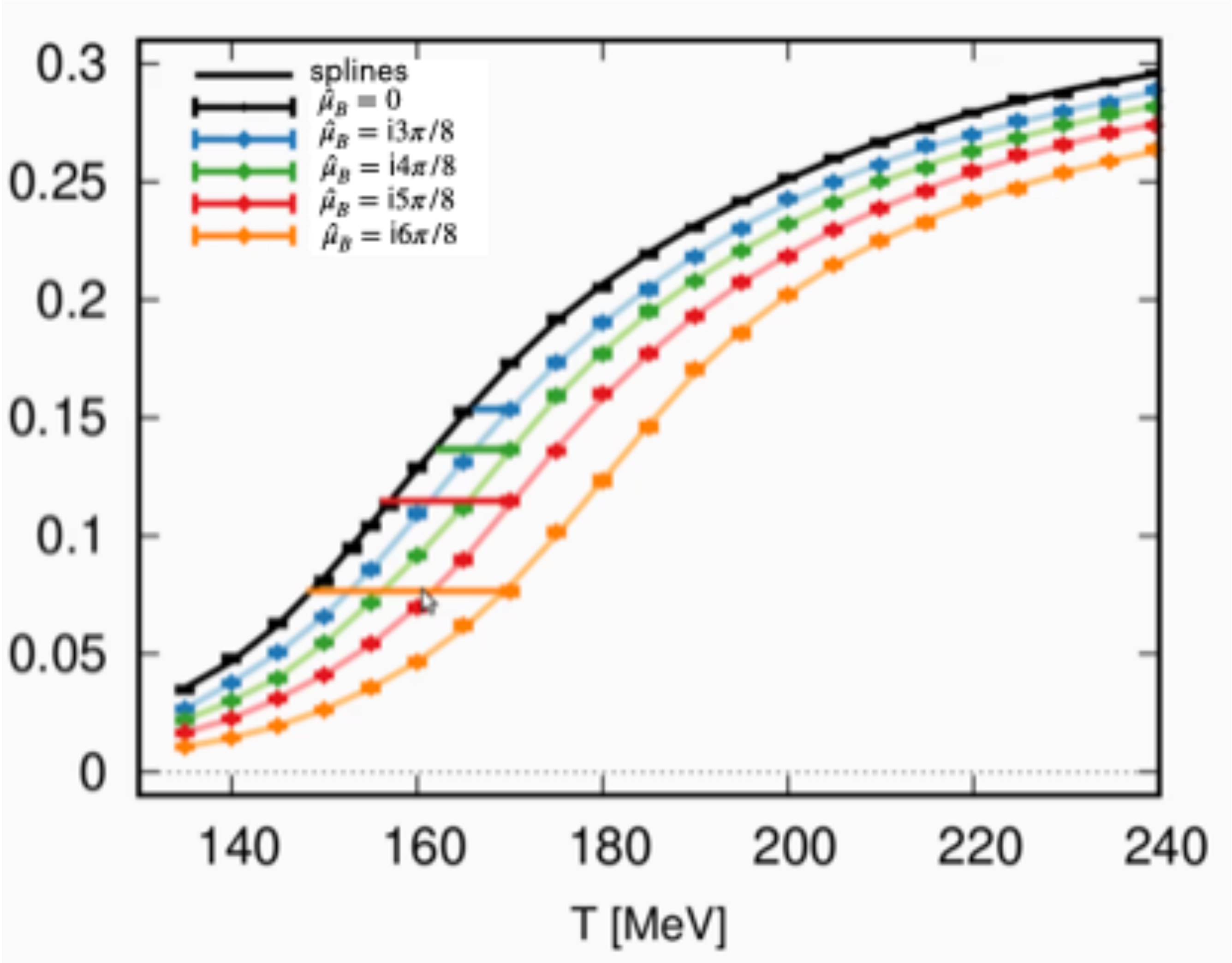
[Borsanyi , S et al Phys.Rev.D 110 (2024) 1, L011501. (2023)]

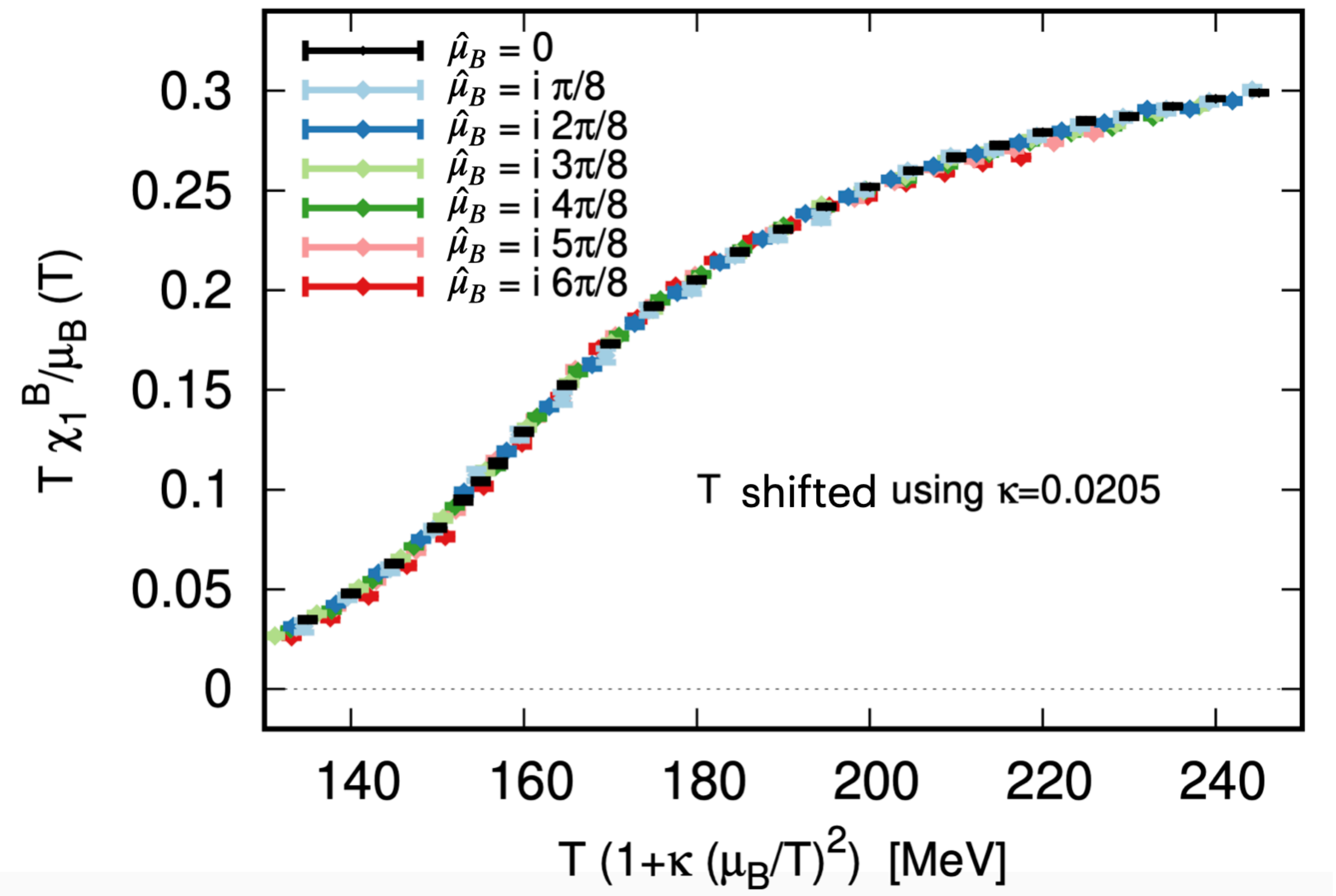


[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]

T′ Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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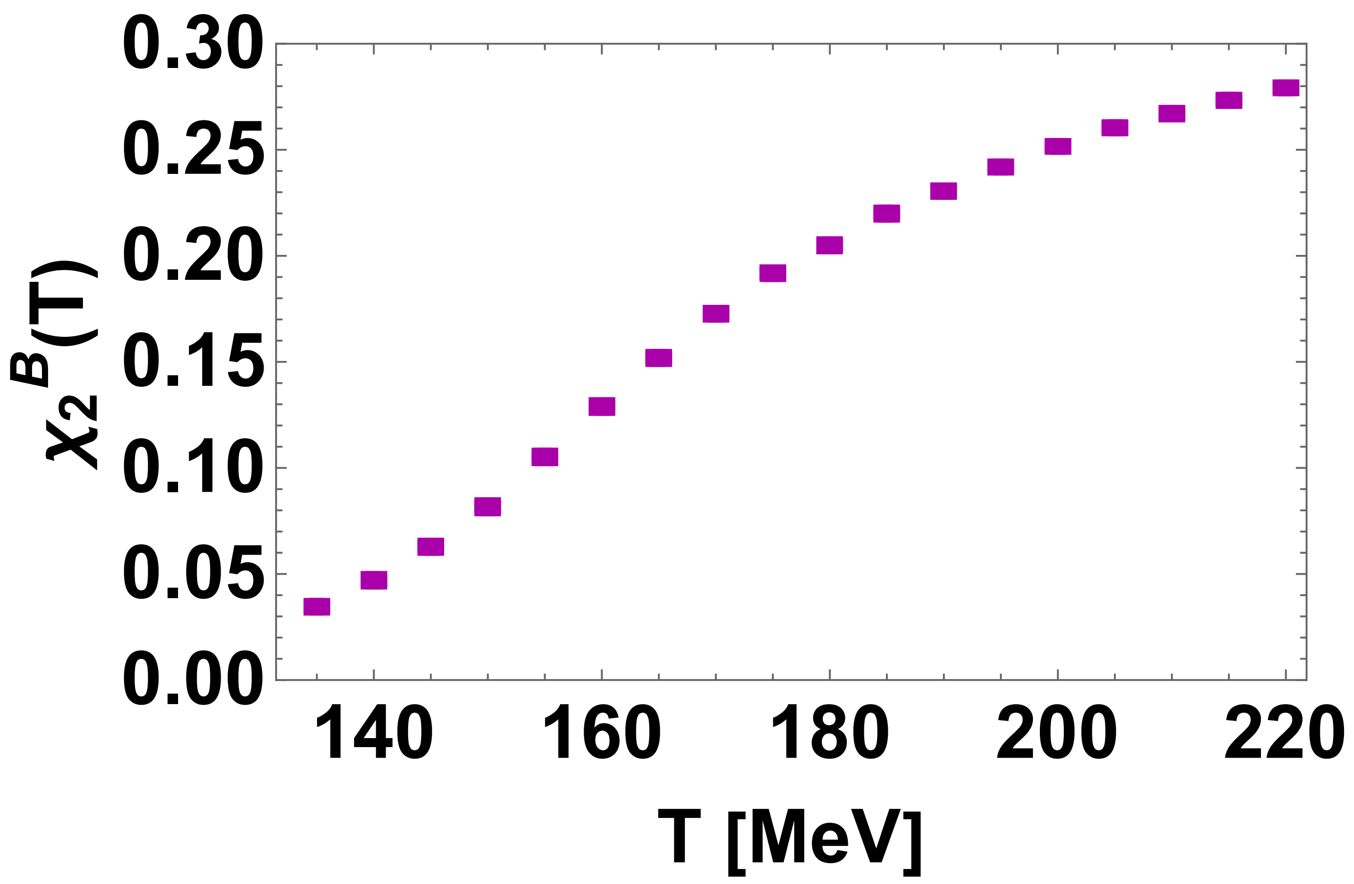


[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

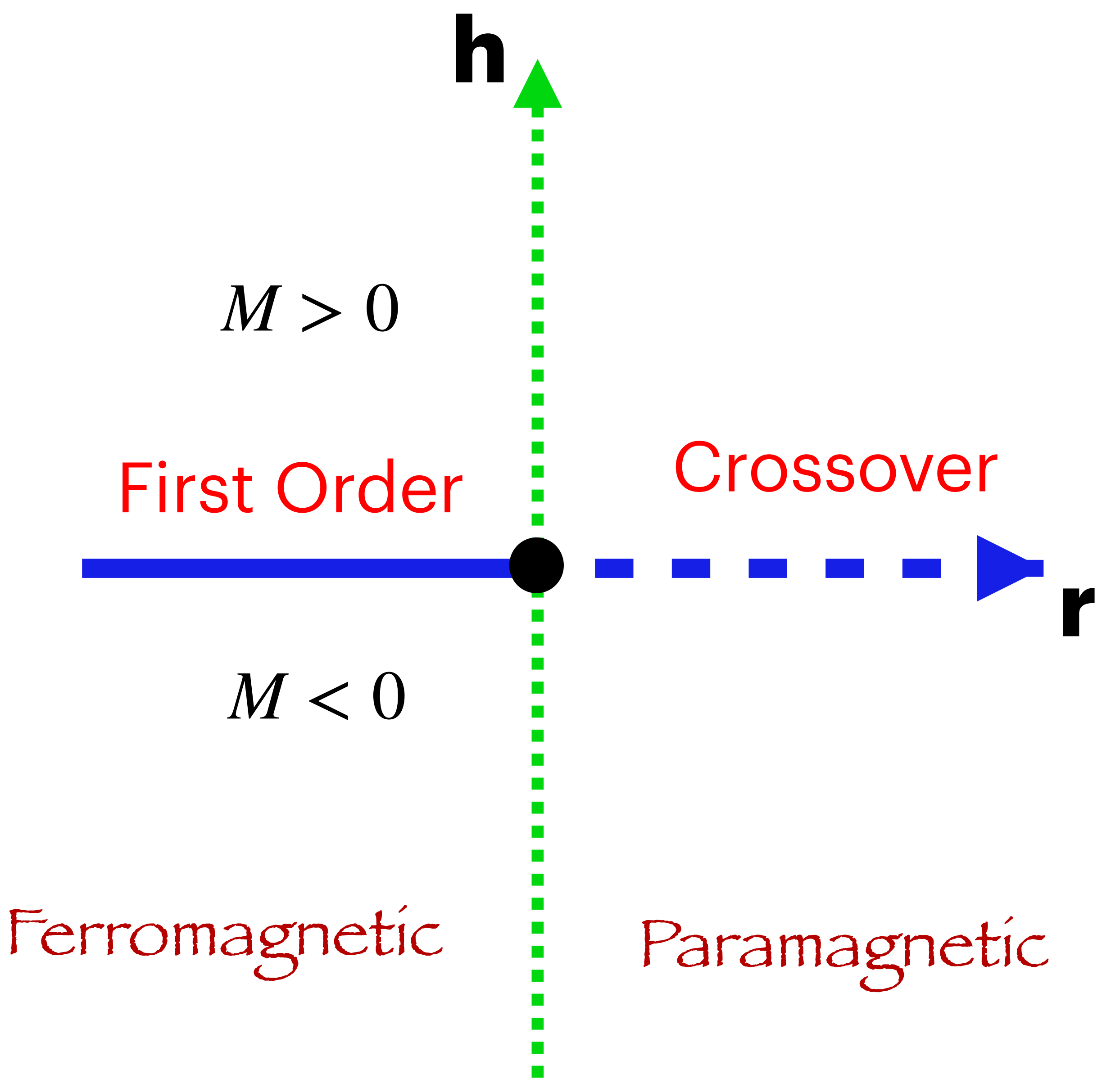
- Uses few expansion terms
- μ_B dependence is captured in T-rescaling.
- Trusted up to $\frac{\mu_B}{T} = 3.5$ in the region where Critical point is expected

See Talk by **Johannes Jahan**
extension to 4D

[M. K et al PhysRevD.109.094046]

Introducing Critical Point

Mapping 3D Ising to QCD

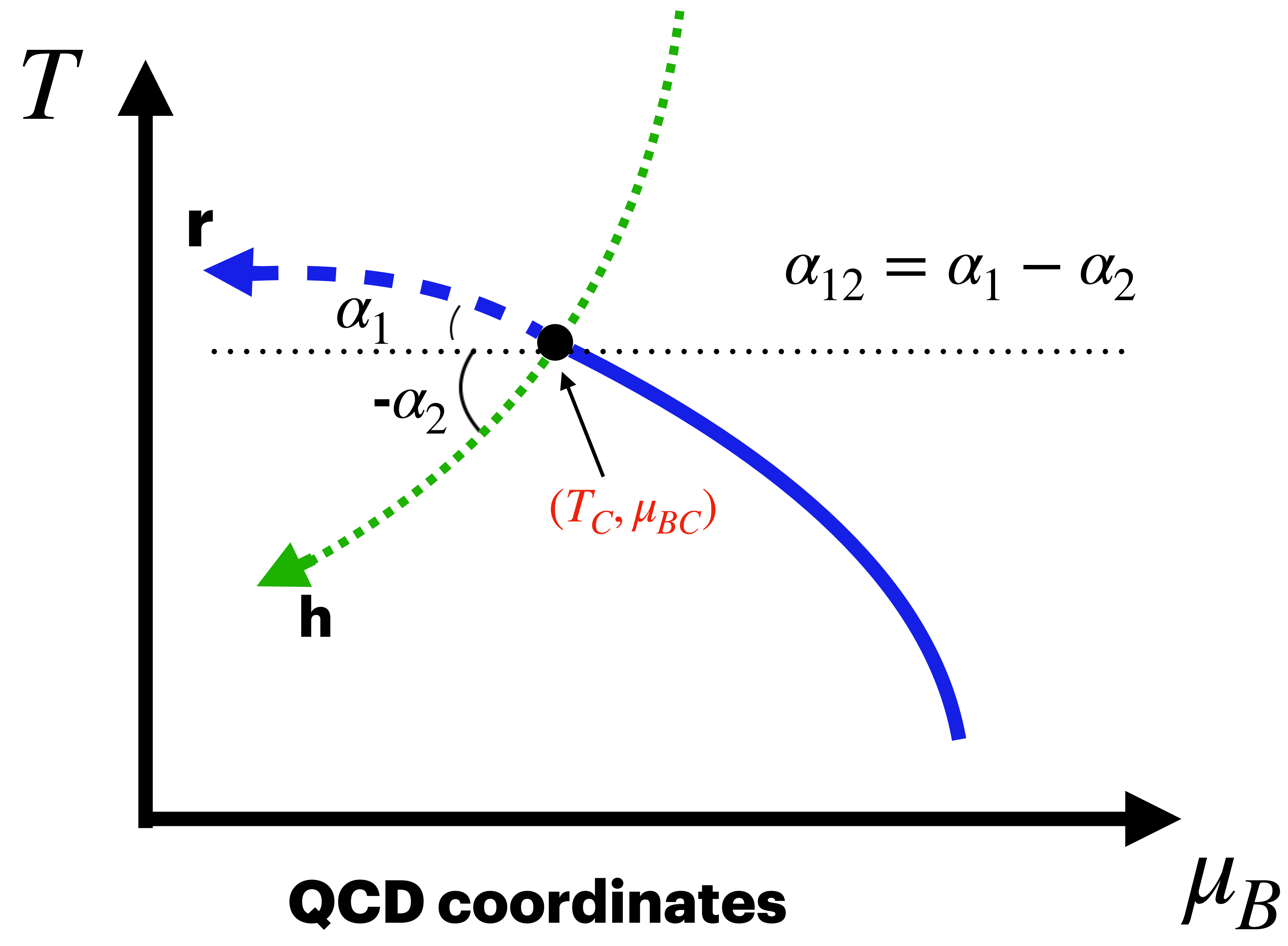


3D Ising coordinates

$$\frac{T'-T_0}{T_C T'_{,T}} = -w' h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w'(-r\rho' - h \cos \alpha'_{12})$$

$$T' = T\left[1 + \left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4\right]$$



QCD coordinates

[M. K et al PhysRevD.109.094046]

- Free parameters μ_{BC}, T_C, w', ρ', α'_{12} can be fixed by the current physics knowledge

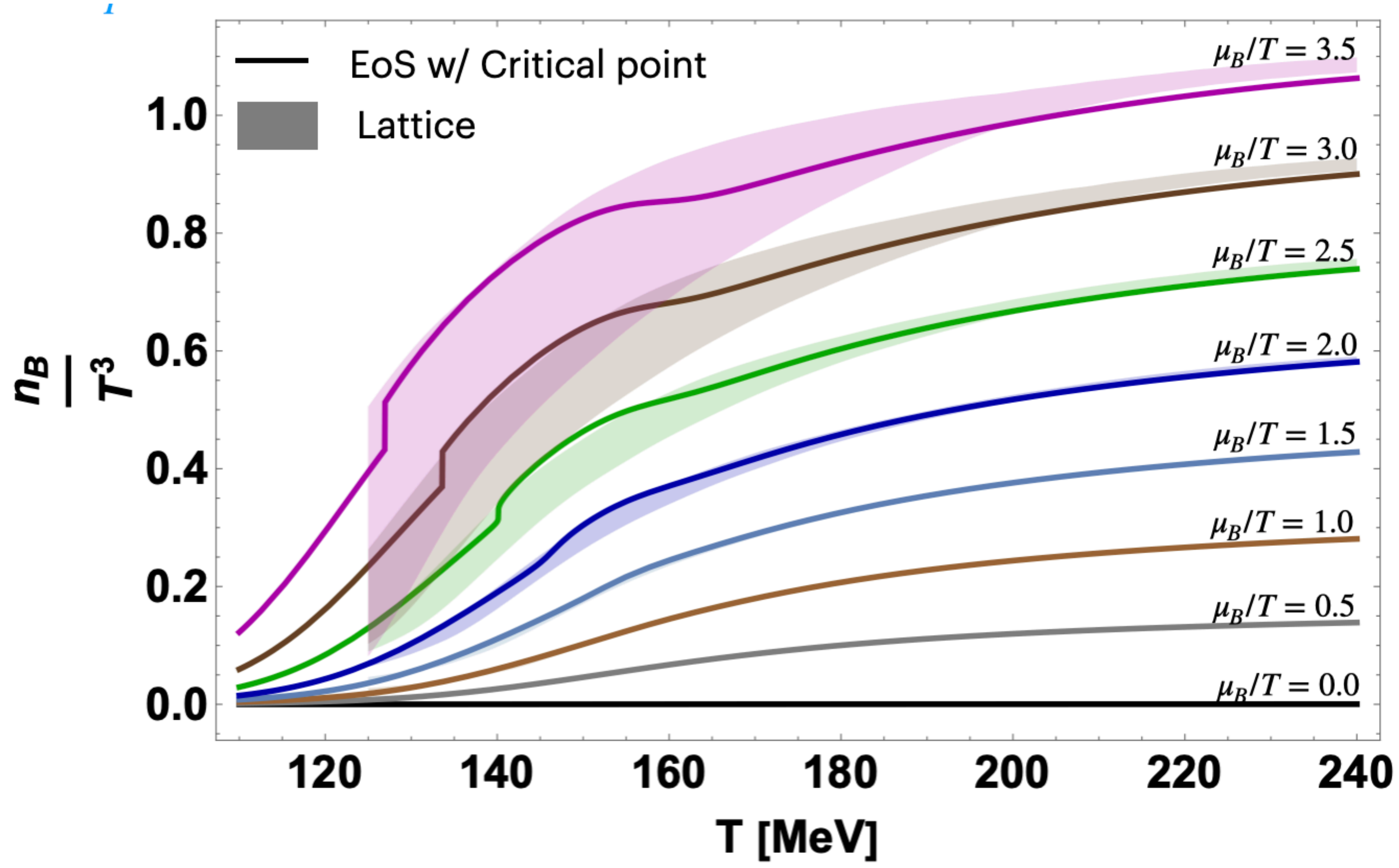
See Talk by **Justin Laberge** constrain parameters using ML

Merging Ising with Lattice (Ising-T ExS)

Full Baryon Density

$$\frac{n_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

$$T' = T'_{Crit}(T,\mu_B) + T'_{Non-Crit}(T,\mu_B)$$



Parameter choice

$\mu_{BC} = 350$ MeV

$T_C = 140$ MeV

$\alpha_{12} = \alpha_1 = 6.6^0$

$\alpha_2 = \alpha_1 - \alpha_{12}$

$w = 2$

$\rho = 2$

[M. K et al PhysRevD.109.094046]

Thermodynamic Observables

Parameter choice

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

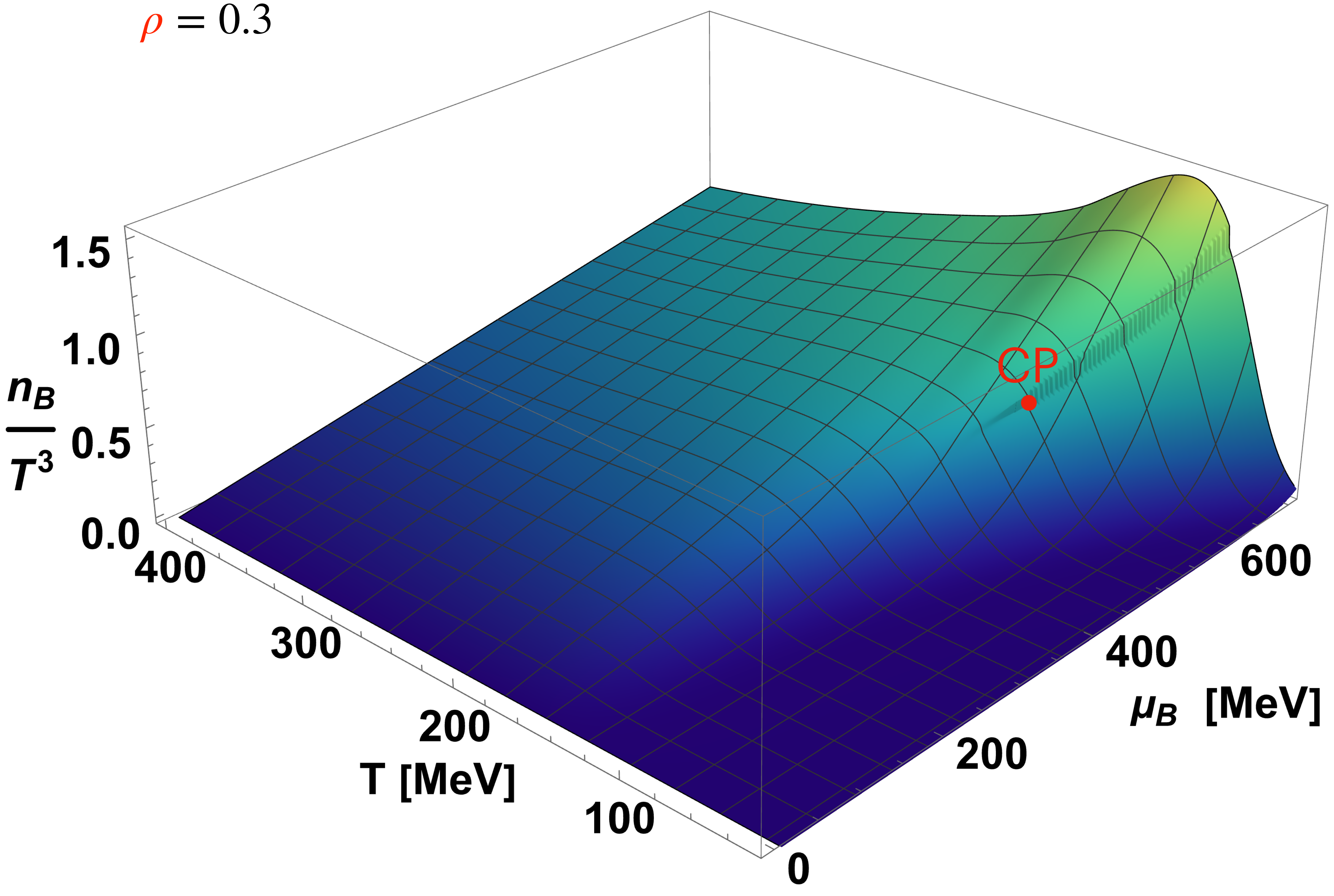
$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

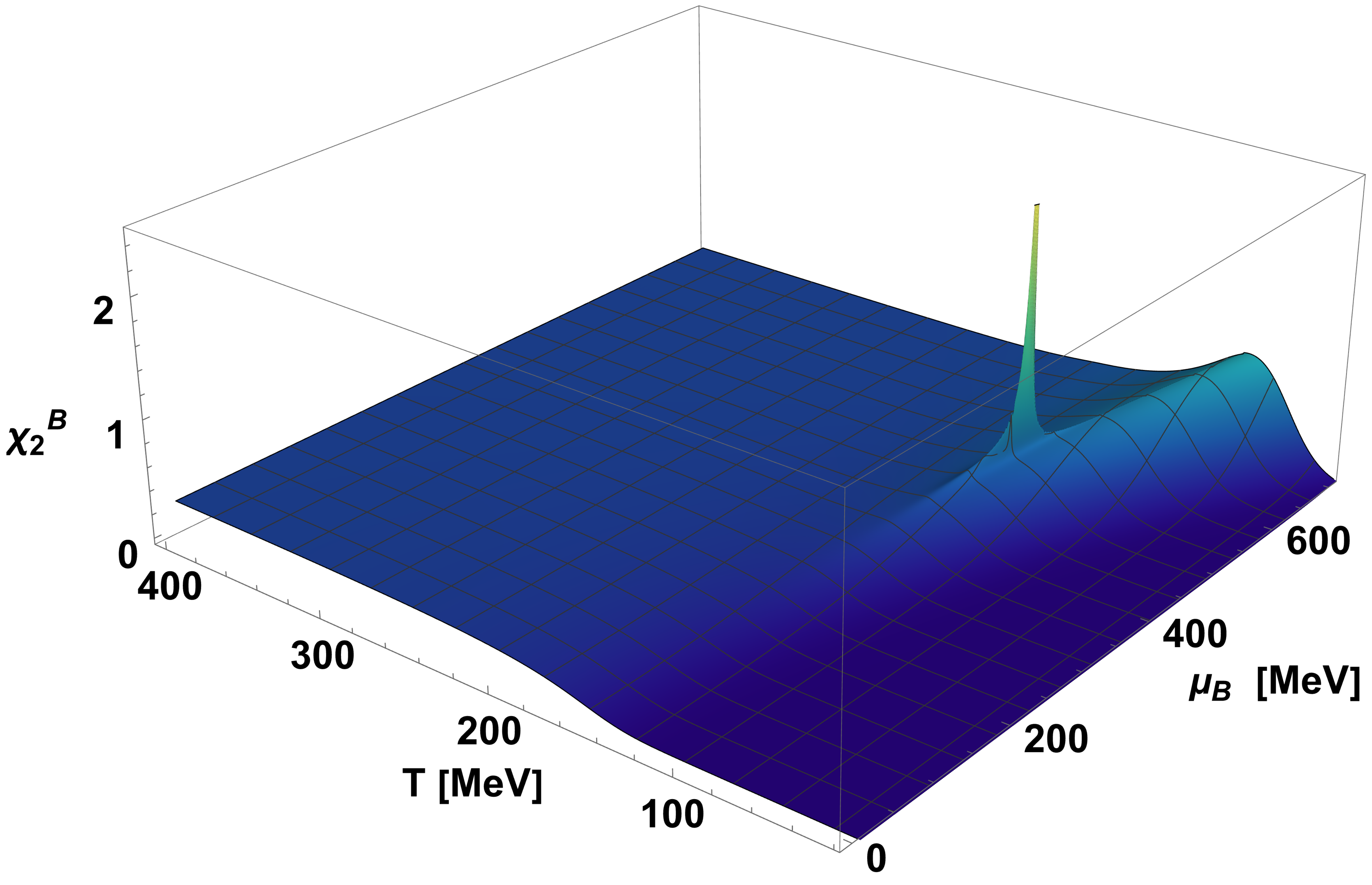
$w = 15$

$\rho = 0.3$

Baryon Density



Baryon number susceptibility



[M. K et al PhysRevD.109.094046]

Summary and Conclusions

- We provide an Equation of State with enhanced coverage with 3D-Ising model Critical Point

DOI 10.5281/zenodo.14637802

(Open Software)

MUSES collaboration cyberinfrasture

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- Our equation of state, has adjustable parameters, and can be used as input in **hydrodynamical simulations** to compare with experimental searches for the **critical point** in Beam Energy Scan II

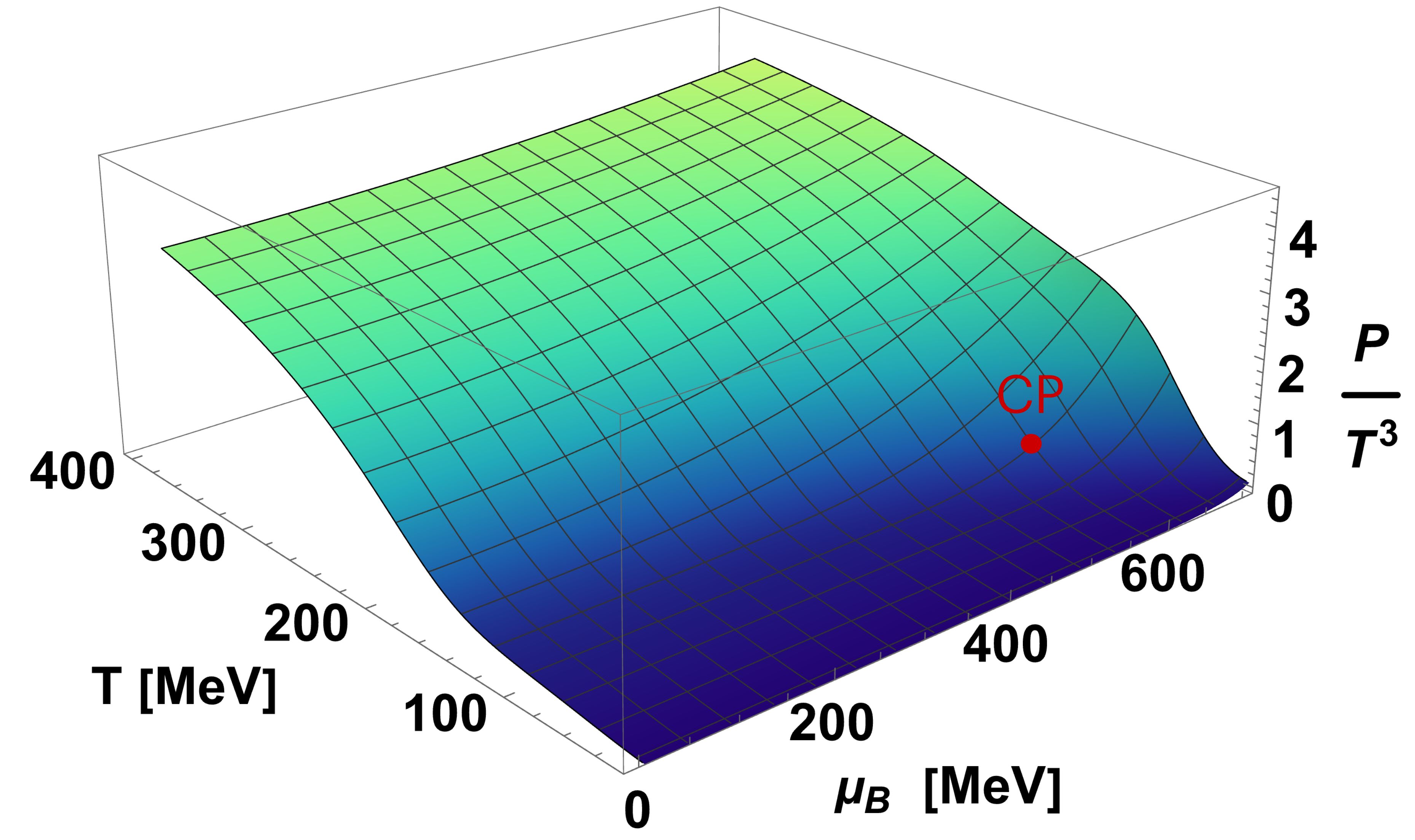
Disclaimer! : We don't predict the location of the critical point

Collaborators:

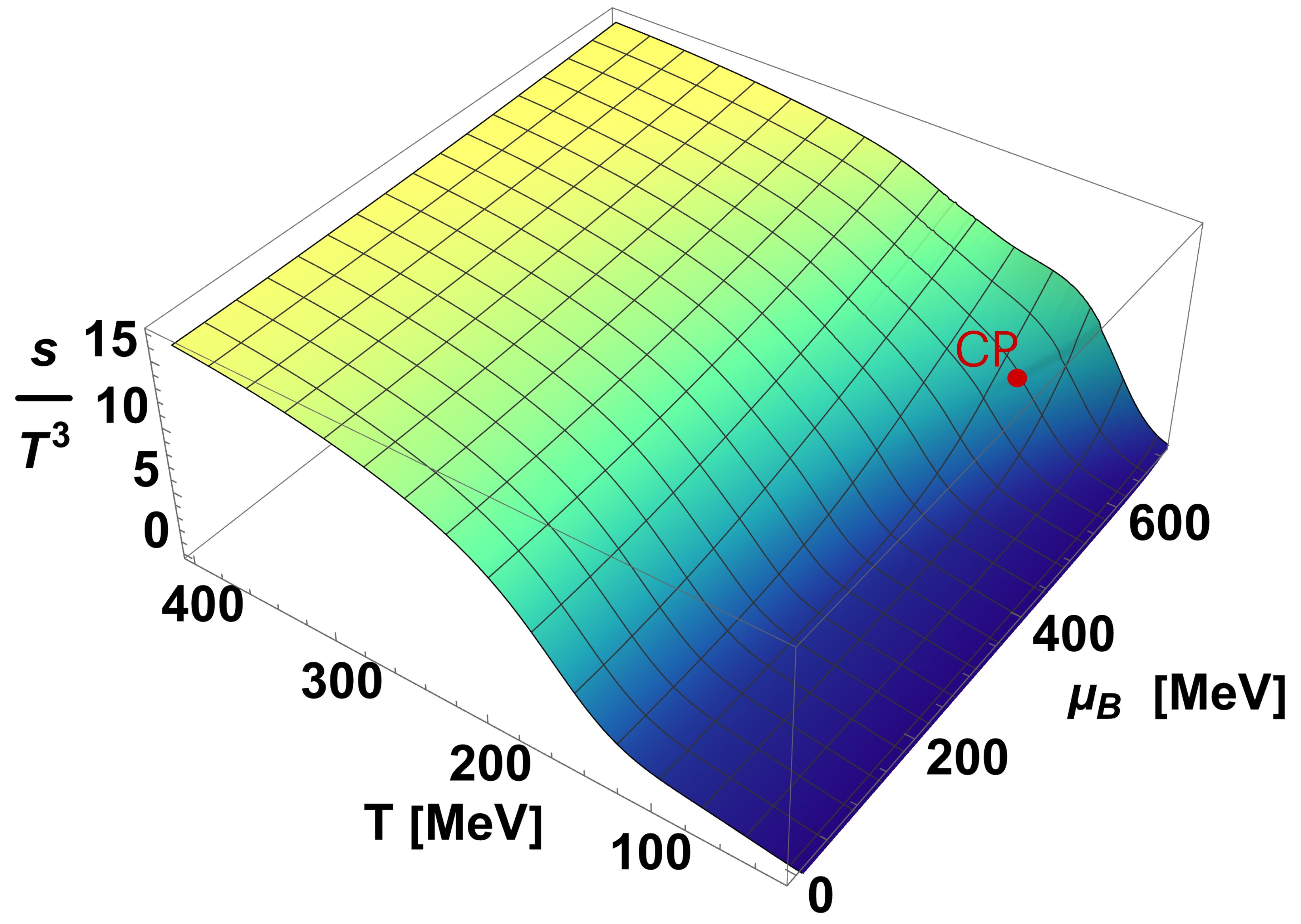
[M K, Steffen A Bass, Elena Bratkovskaya, Johannes Jahan, Pierre Moreau, Paolo Parotto, Damien Price, Claudia Ratti, Olga Soloveva, Mikhail Stephanov, Irene Gonzalez, Jorge A Muñoz, Volodymyr Vovchenko]

Thermodynamic Observables

Pressure



Entropy density



Parameter choice

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

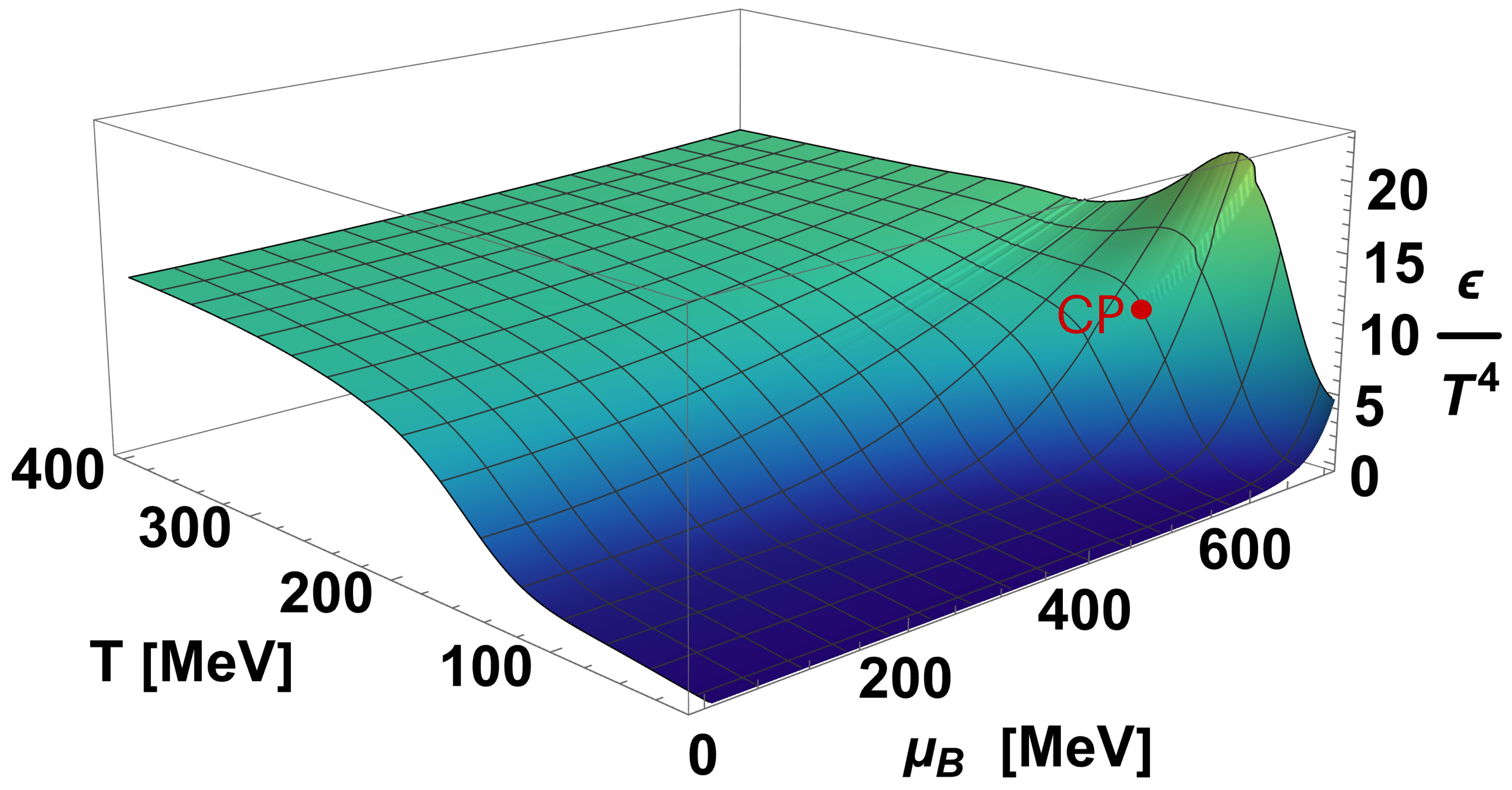
$\alpha_{12} = \alpha_1 = 14^0$

$\alpha_2 = 0^0$

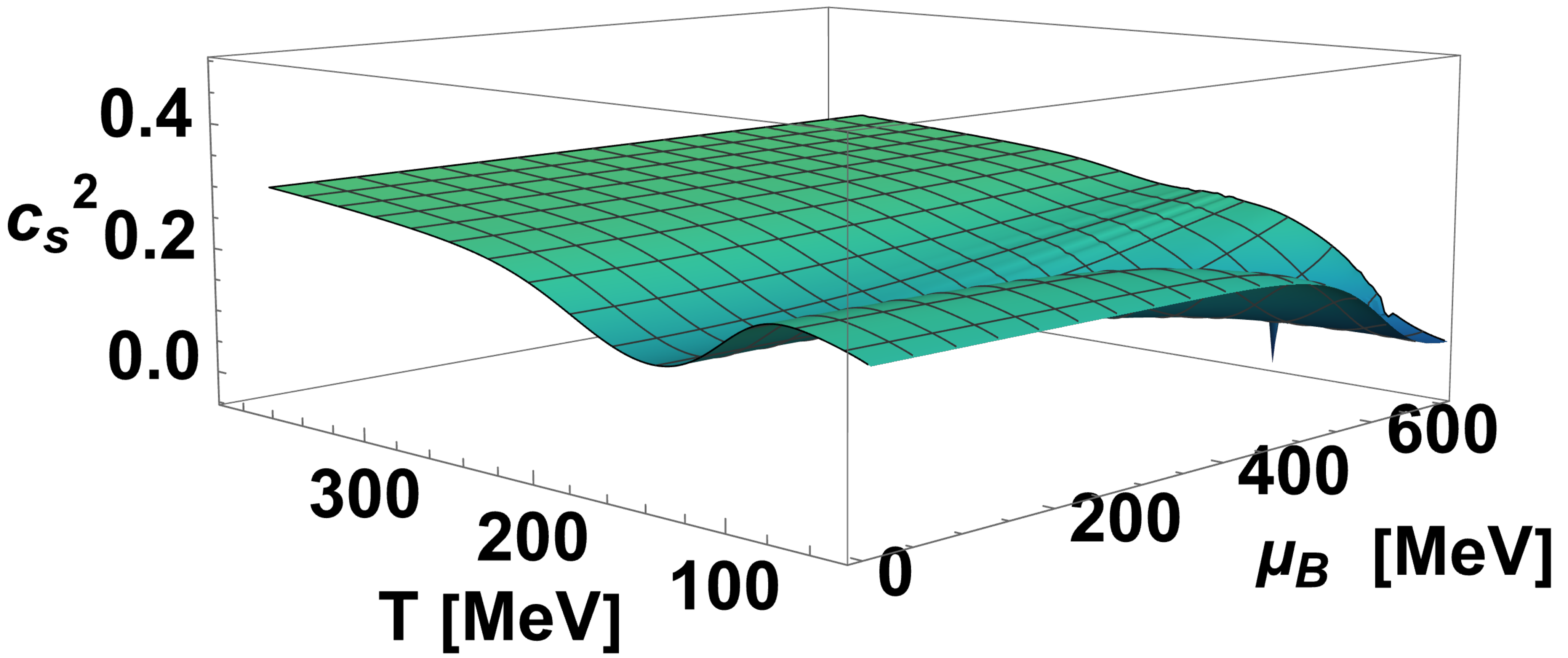
$w = 15$

$\rho = 0.3$

Energy Density



Speed of Sound



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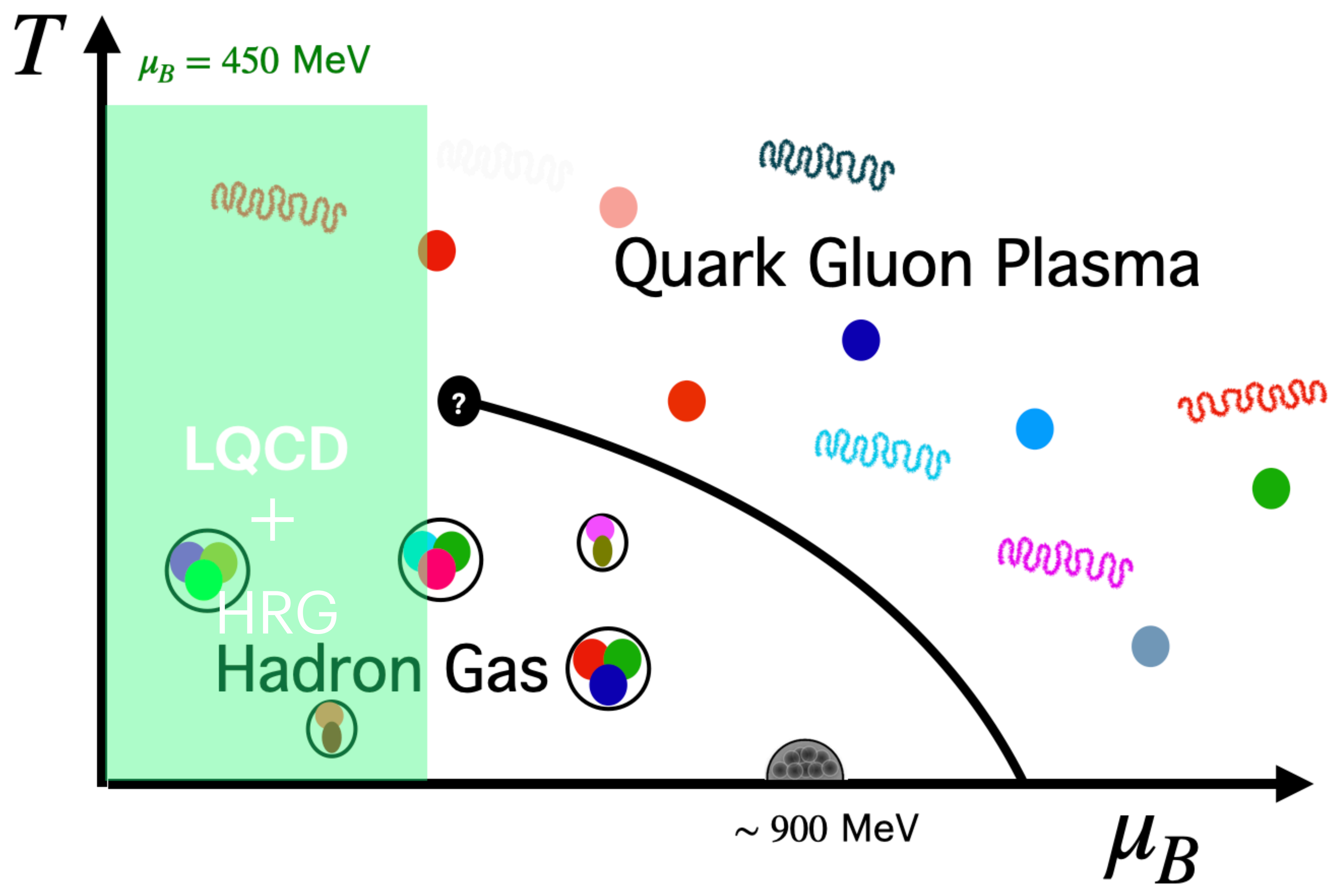
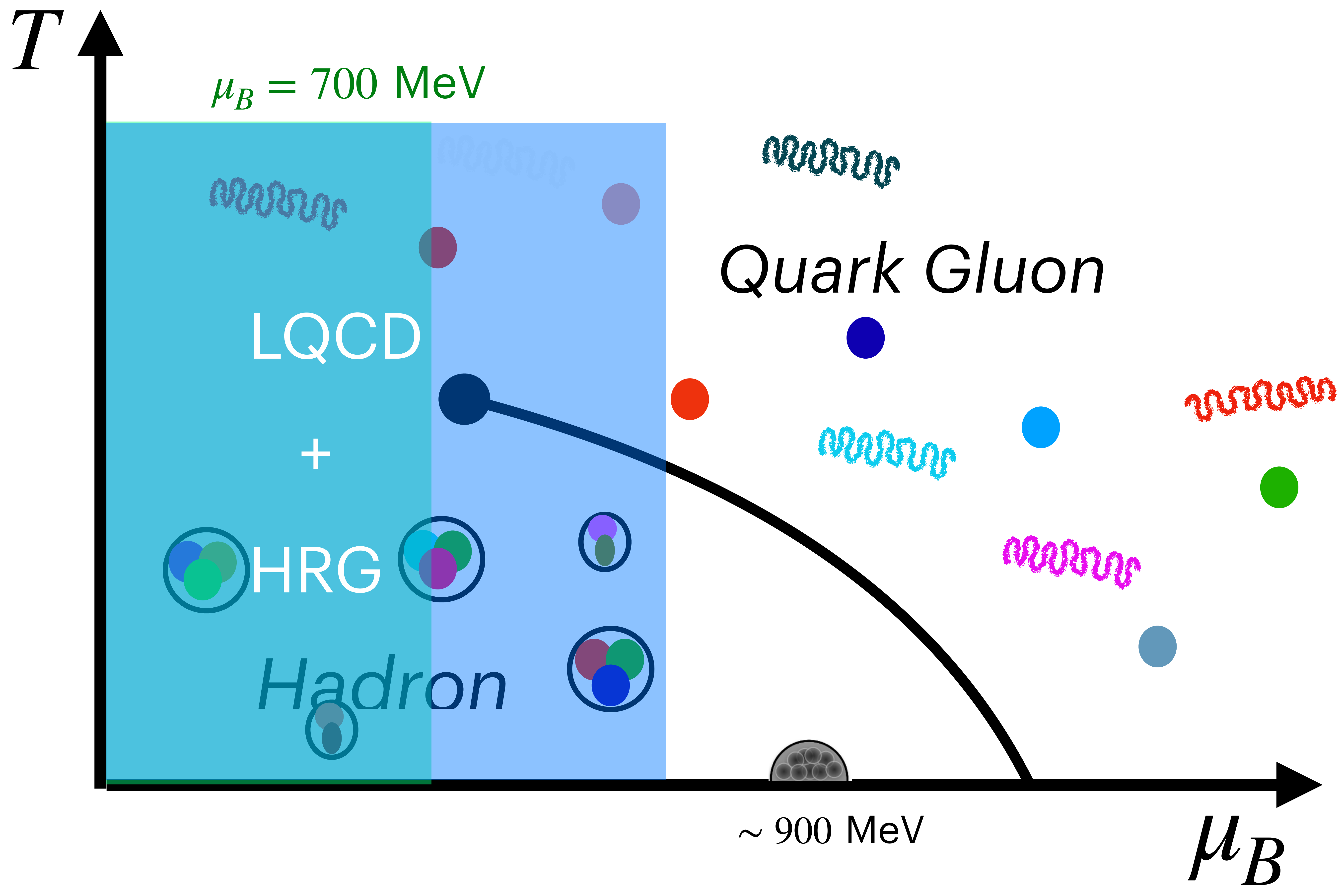
Taylor Expansion



T′ Expansion



Important relations

Relationship with BEST collaboration EoS

- The mapping is not universal
- Quadratic mapping is related to BEST Collaboration (linear) mapping

$$\mu_{BC}, T_C, \alpha'_{12}, w', \rho' \longrightarrow \mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$$

6 parameters

Transition Line

$$T'[T_C, \mu_{BC}] = T_0$$

$T_0 = 158$ MeV - crossover temperature at $\mu_B = 0$

Choosing μ_{BC} fixes T_C and α_1

Slope

$$\alpha_1 = \tan^{-1}\left(\frac{2\kappa_2(T_C)\mu_{BC}}{T_C T'_{,T}}\right)$$

Examples

- $\mu_{BC} = 350$ MeV, $T_C = 140$ MeV and $\alpha_1 = 6.6^0$
- $\mu_{BC} = 600$ MeV, $T_C = 94.3$ MeV and $\alpha_1 = 14^0$

BEST COLLABORATION

TExS

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]