Equation of state of strong interactions: Expansion schemes and Critical point

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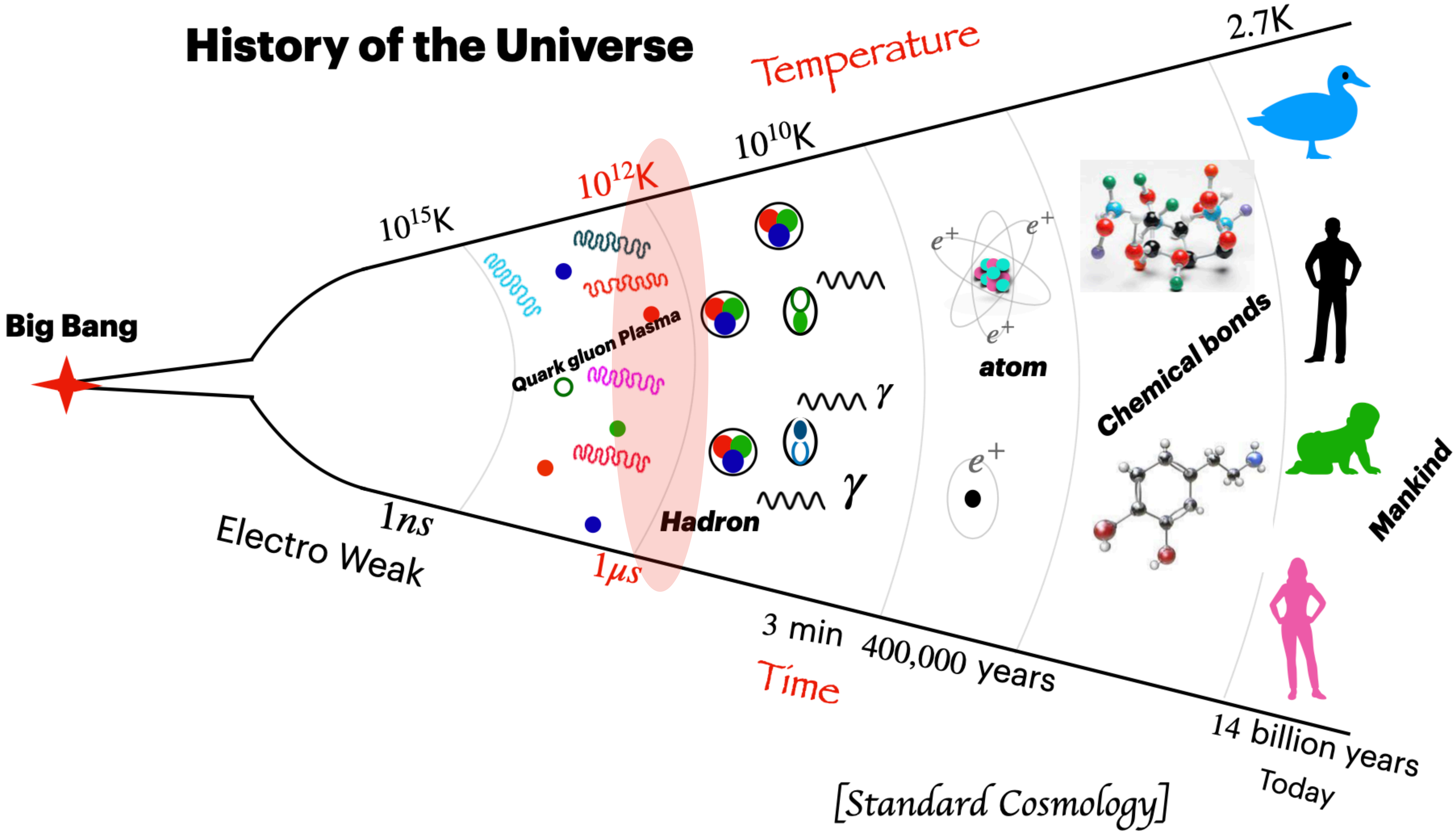
History of the Universe



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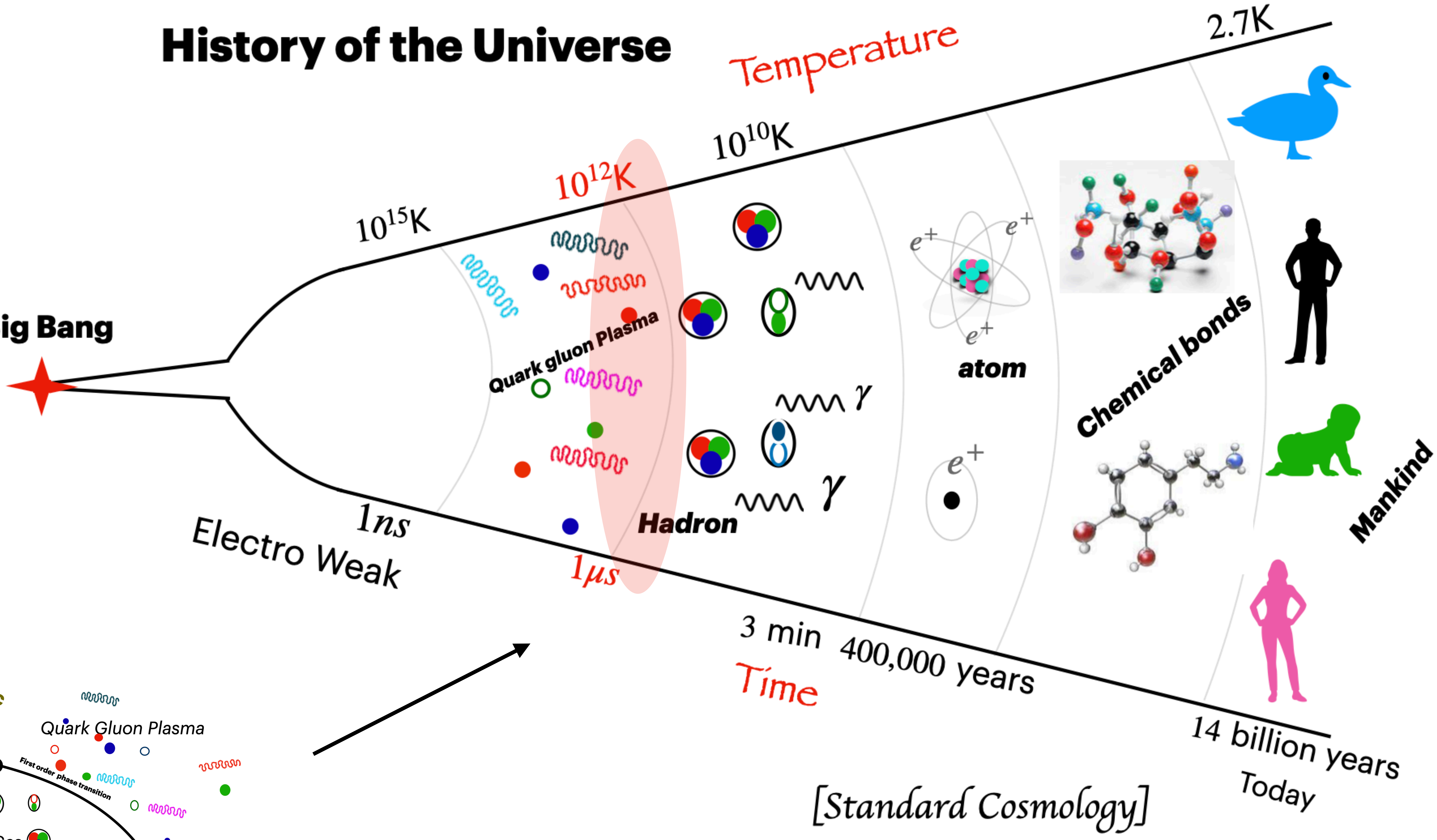
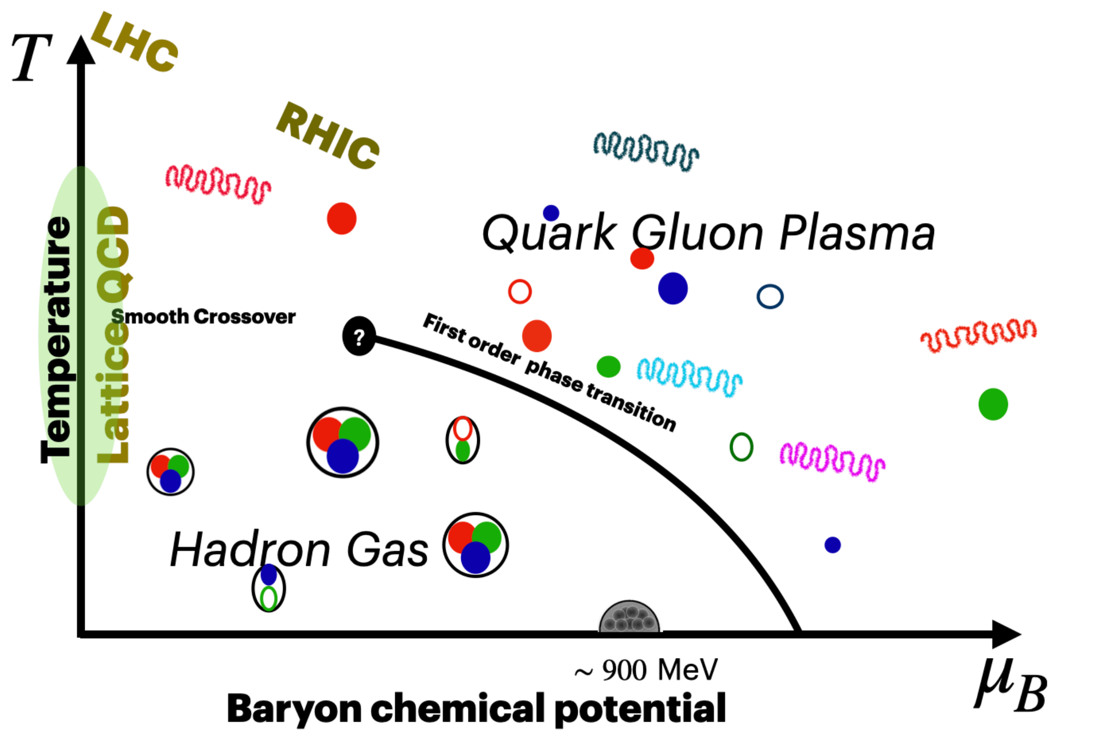
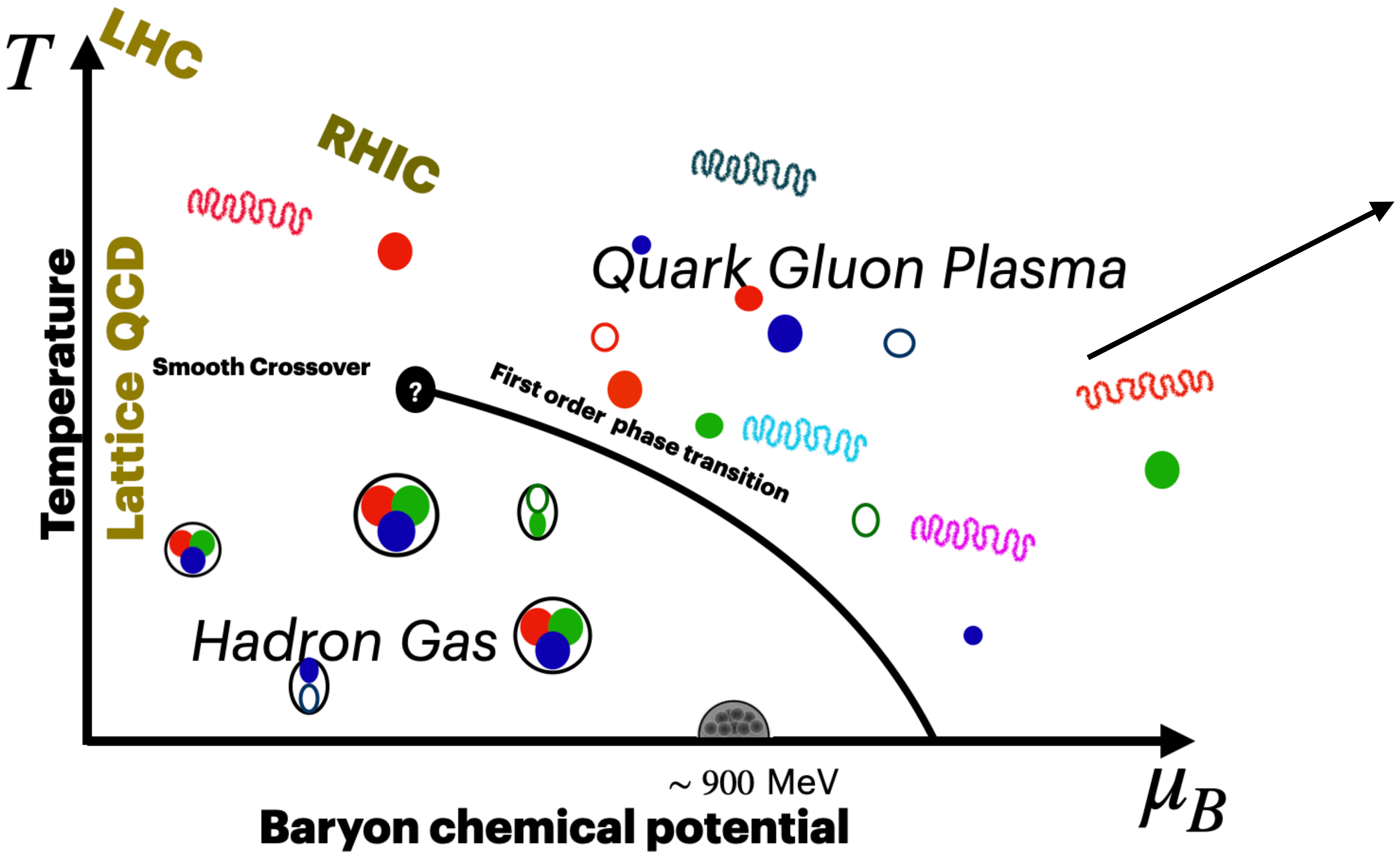
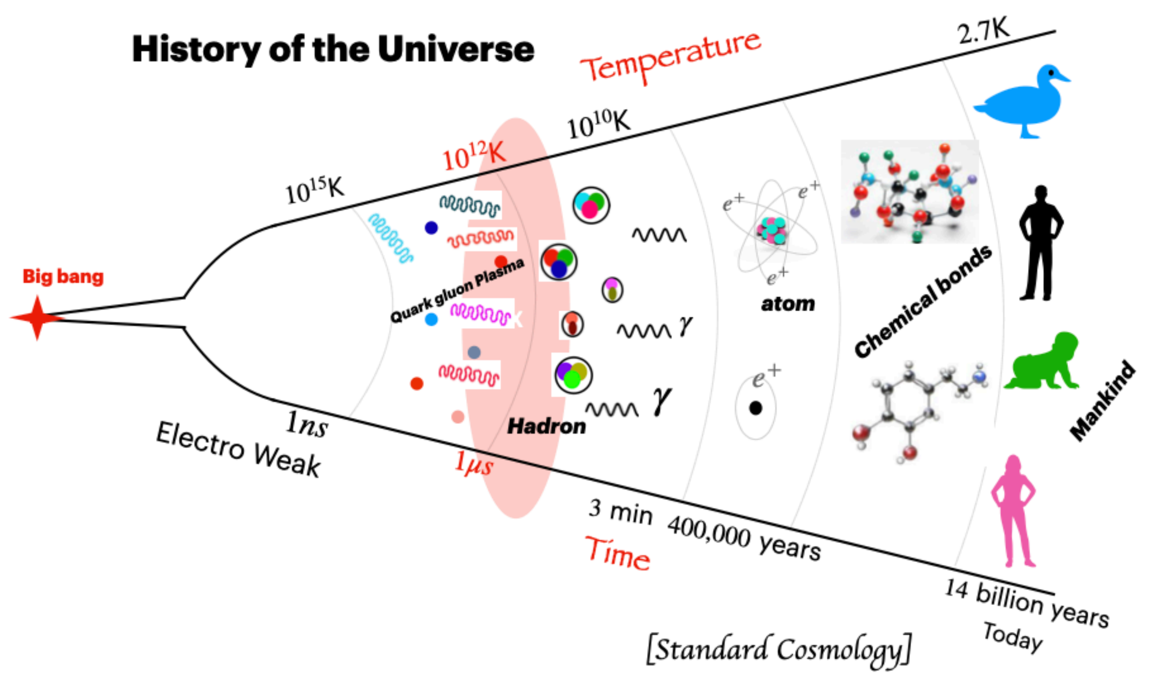




QCD Phase Diagram





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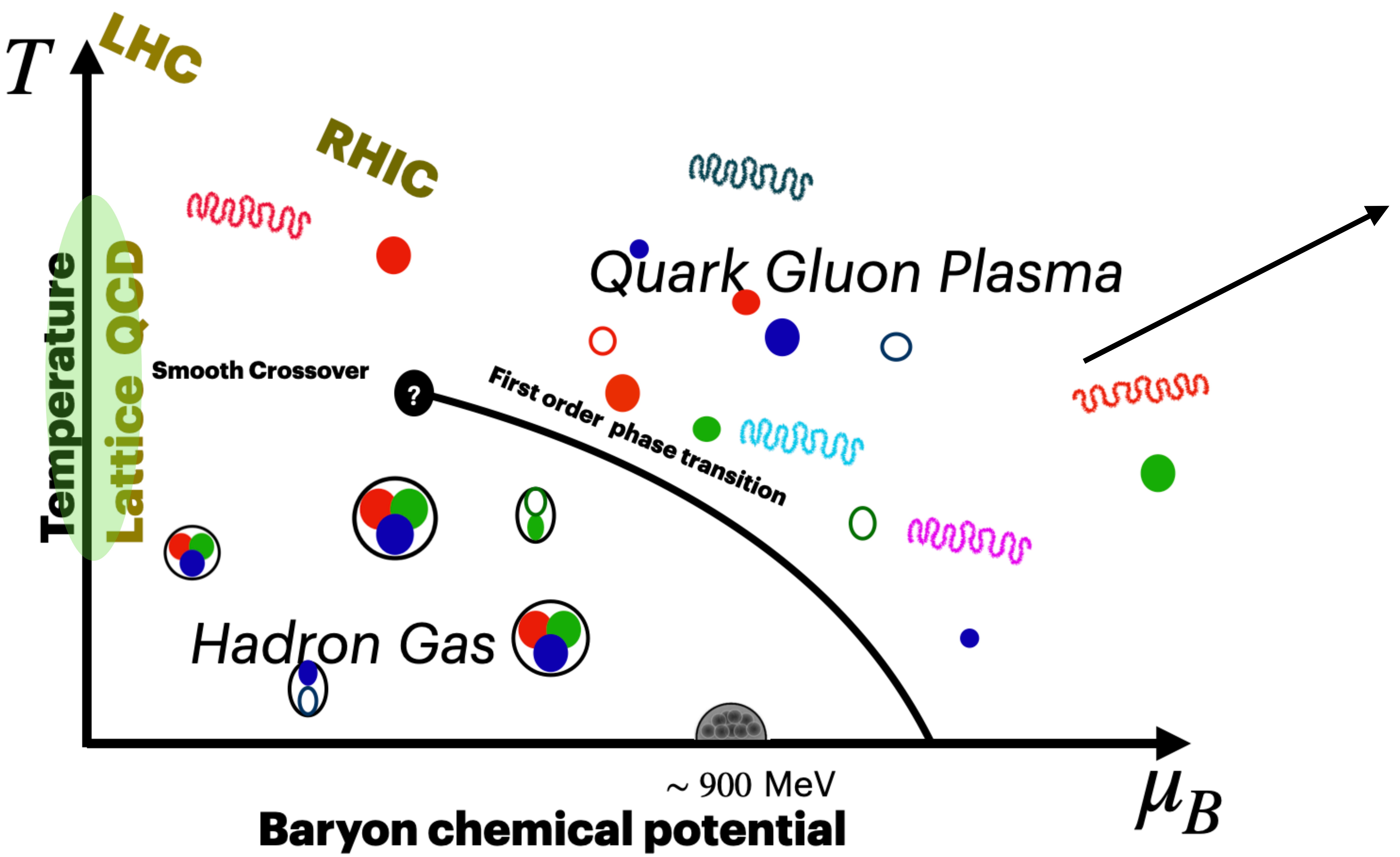

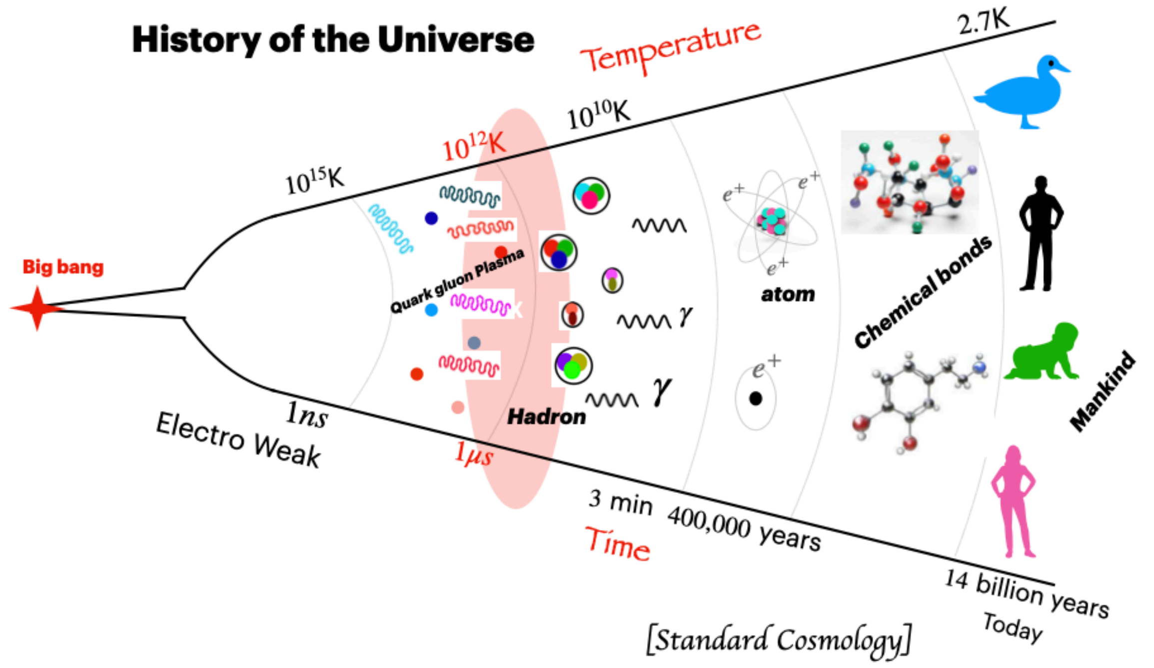

What we know

- At $\mu_B = 0$, de-confinement transition is well established (**Smooth crossover**)
  [Aoki et al Nature. (2006)]
- At finite μ_B, QCD **critical point** is expected but not yet seen
- Lattice simulations are challenging at Finite density **(Fermi-sign problem)**

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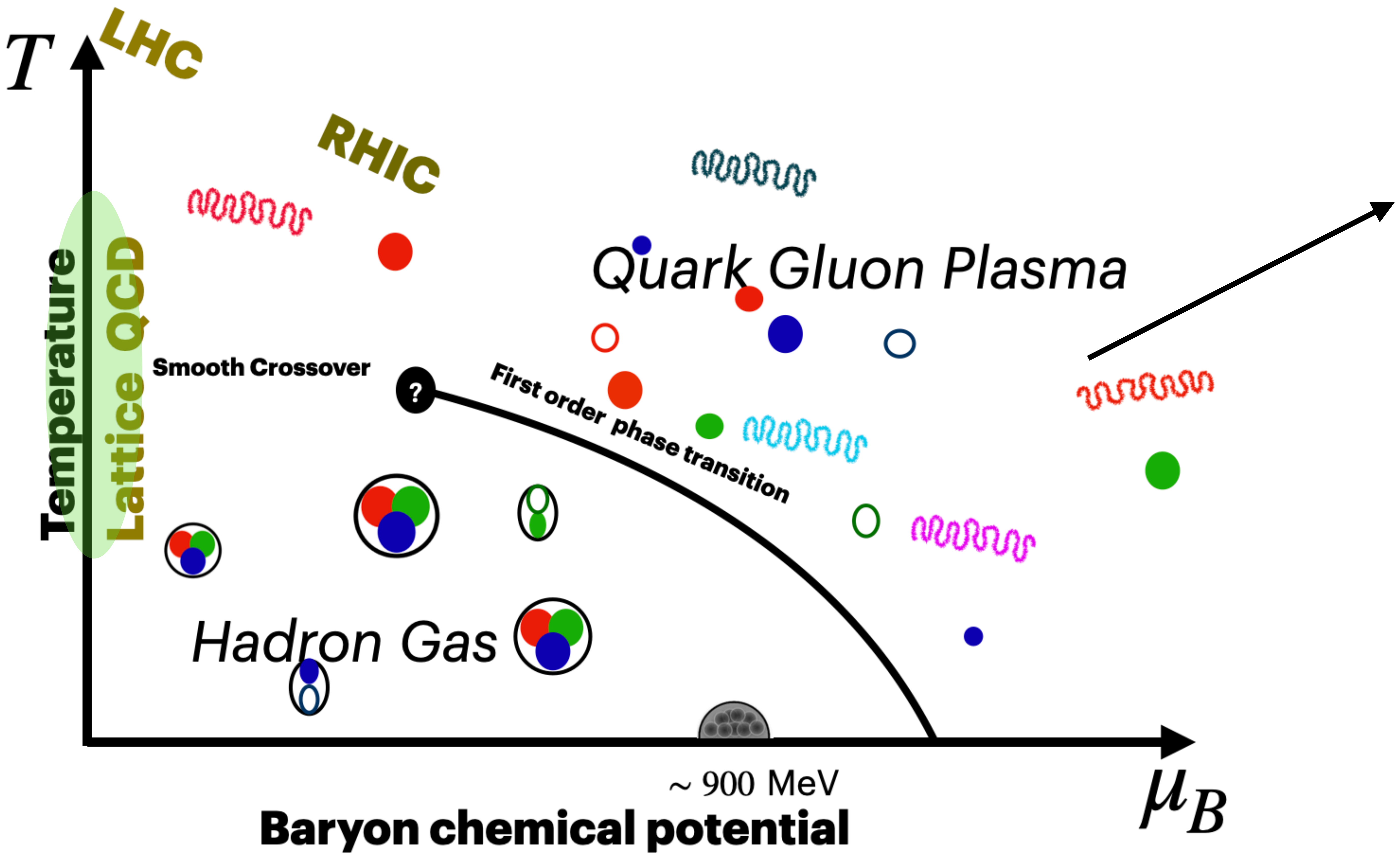

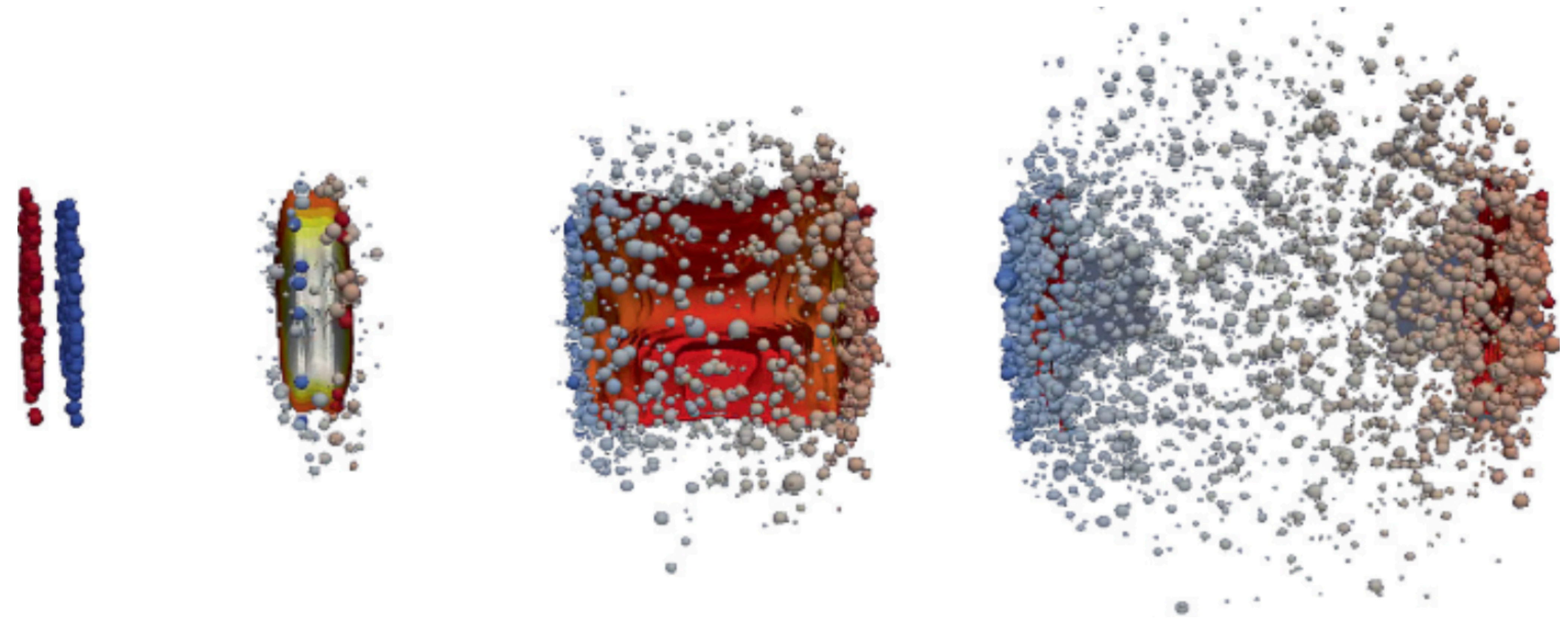

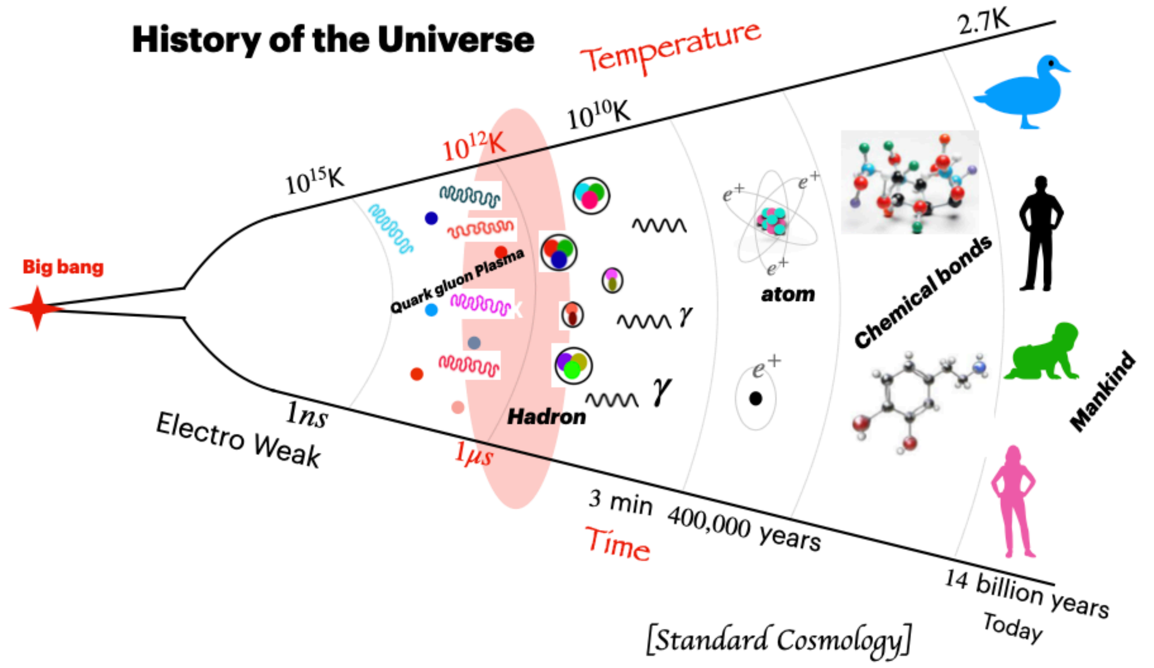

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Attempts

- Experimental programs RHIC,LHC
- **Expansion schemes** are used for finite density physics

Taylor: Lattice QCD results

Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!}\chi_{2n}(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^{2n}$$

[Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]
[Bazavov, A et al PhysRevD.95, 054504 (2017)]

$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\bigg|_{\mu_B=0}$$

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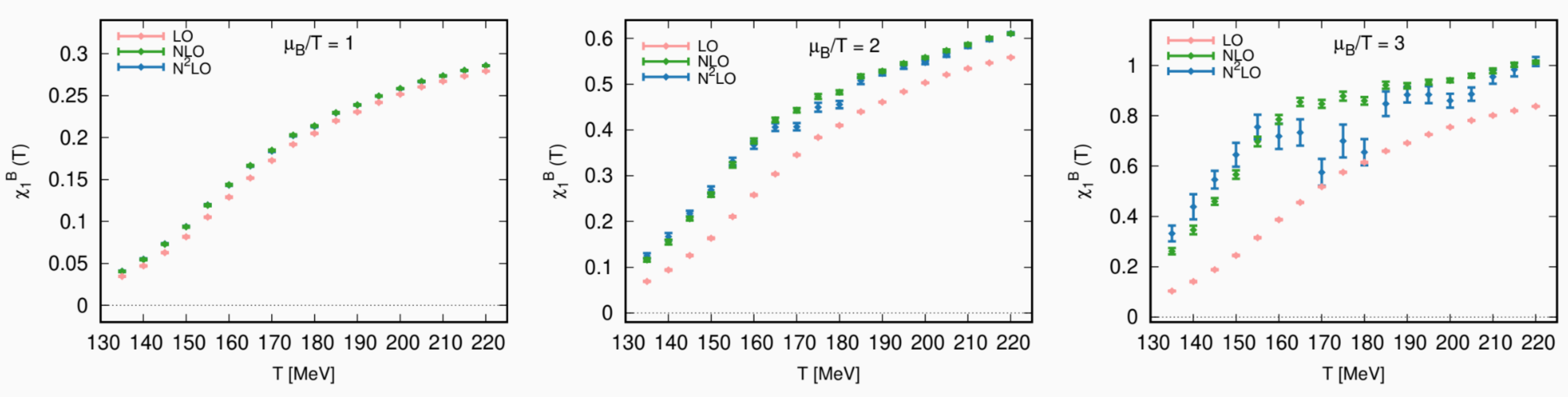
Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational effort**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]

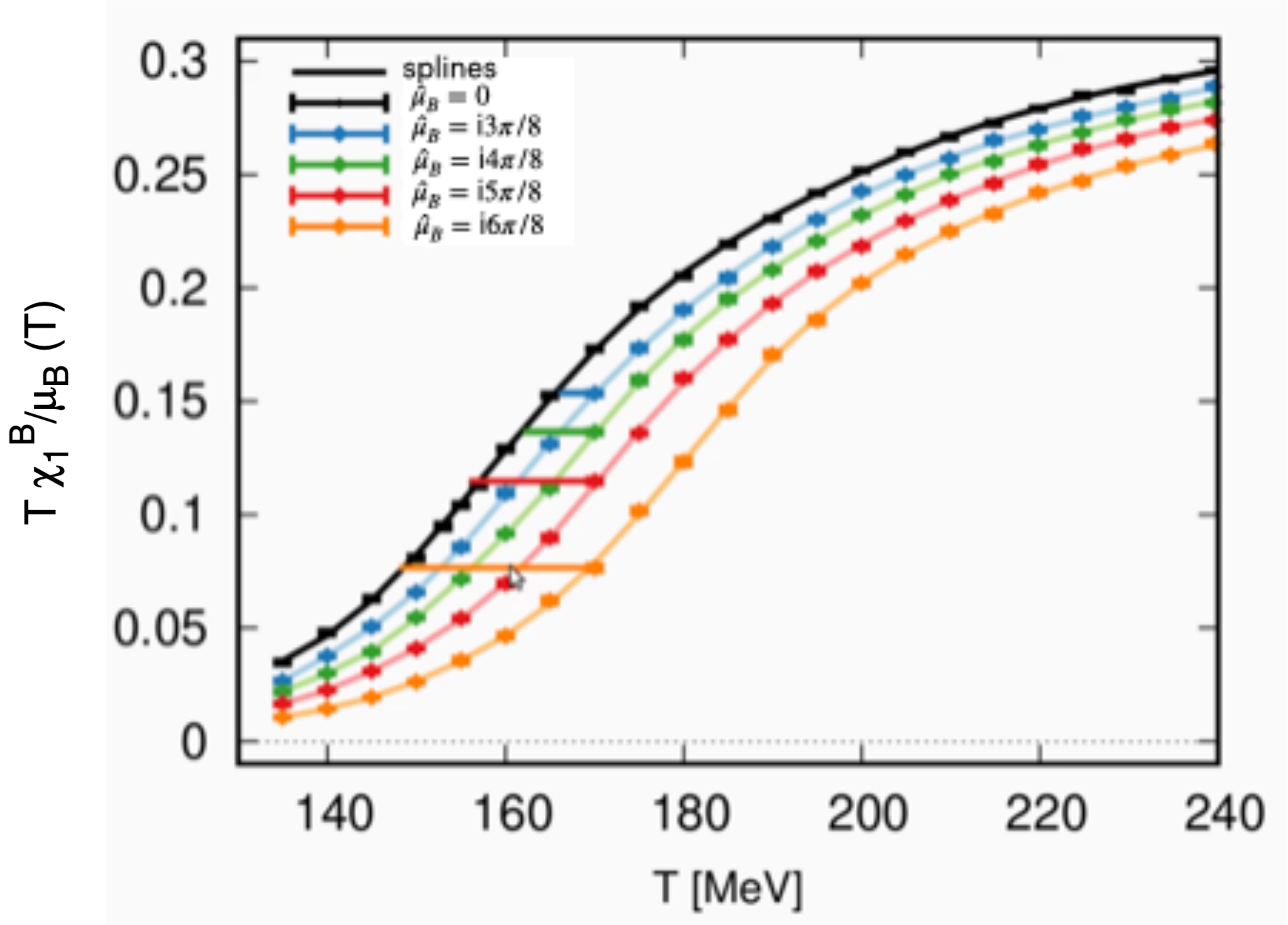
[Borsanyi , S et al arXiV:2312.07528v1. (2023)]

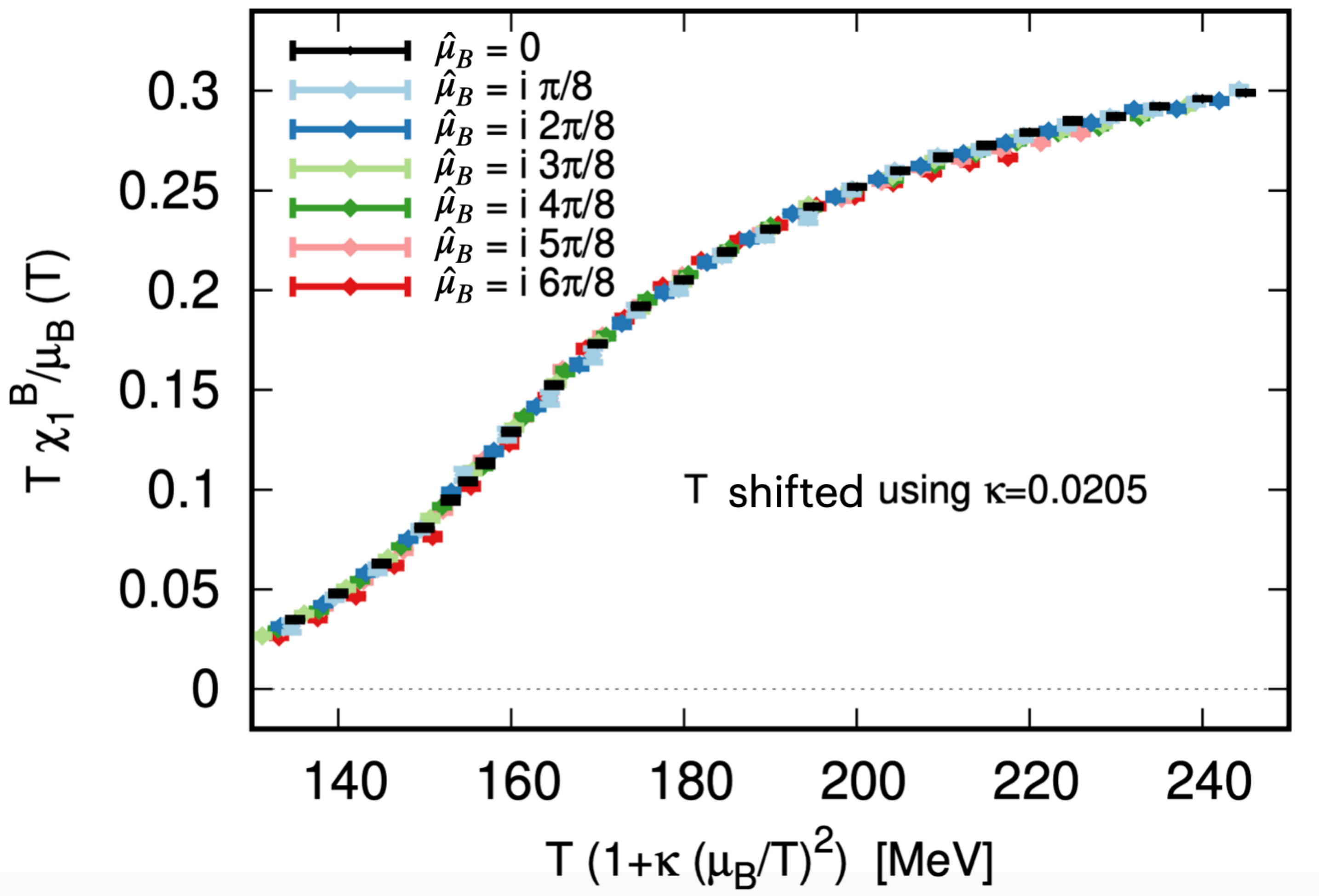


[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]

T′ Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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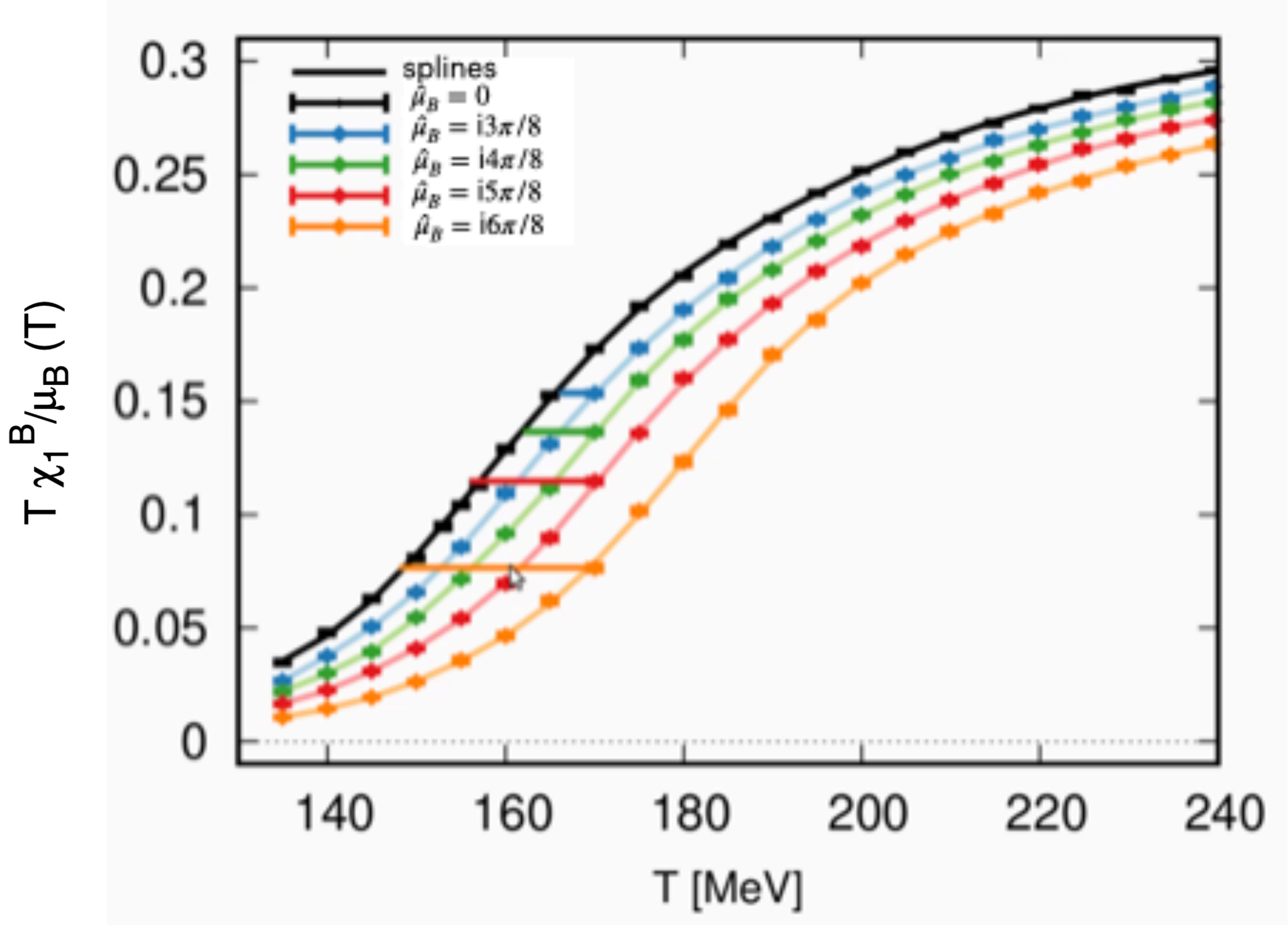

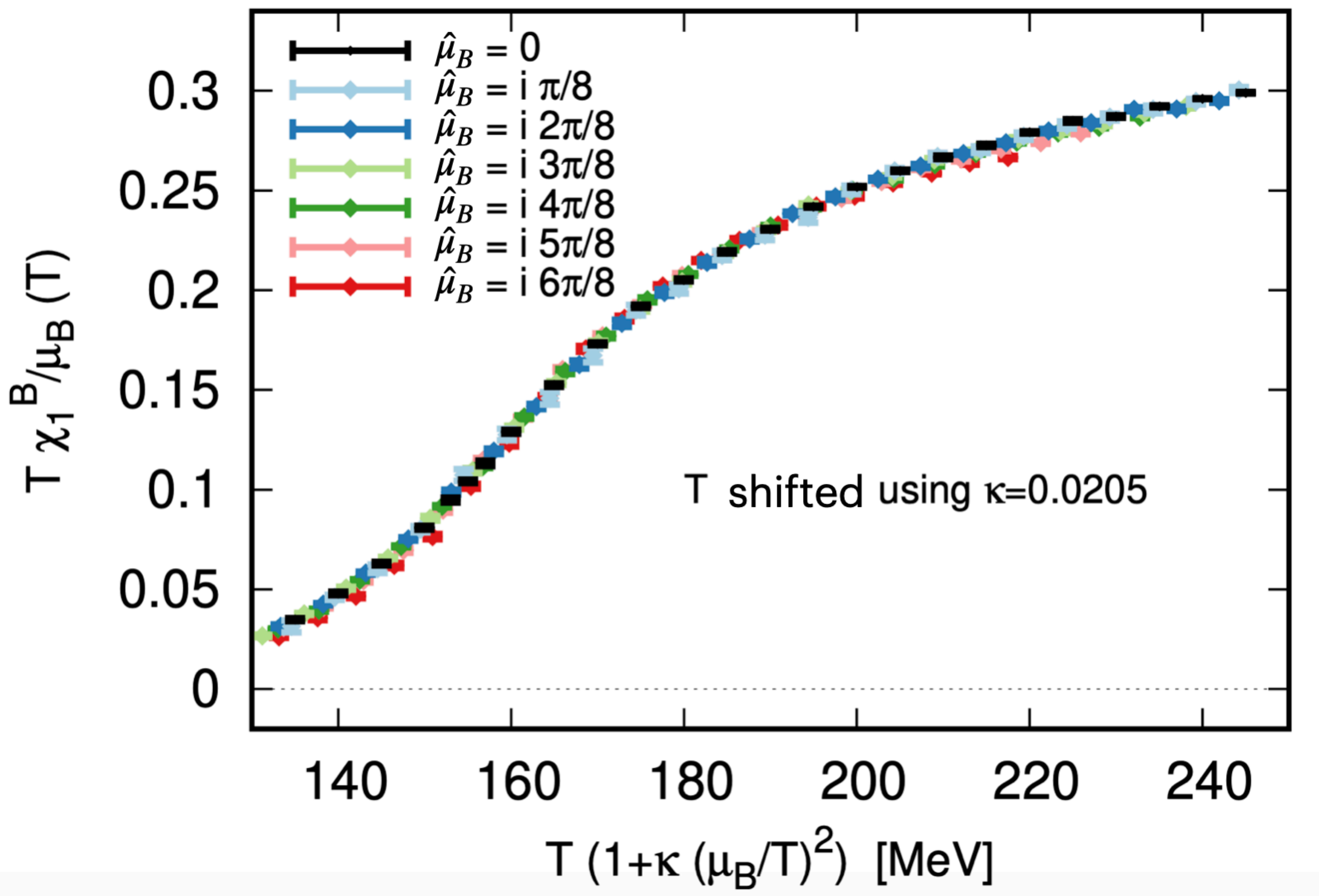

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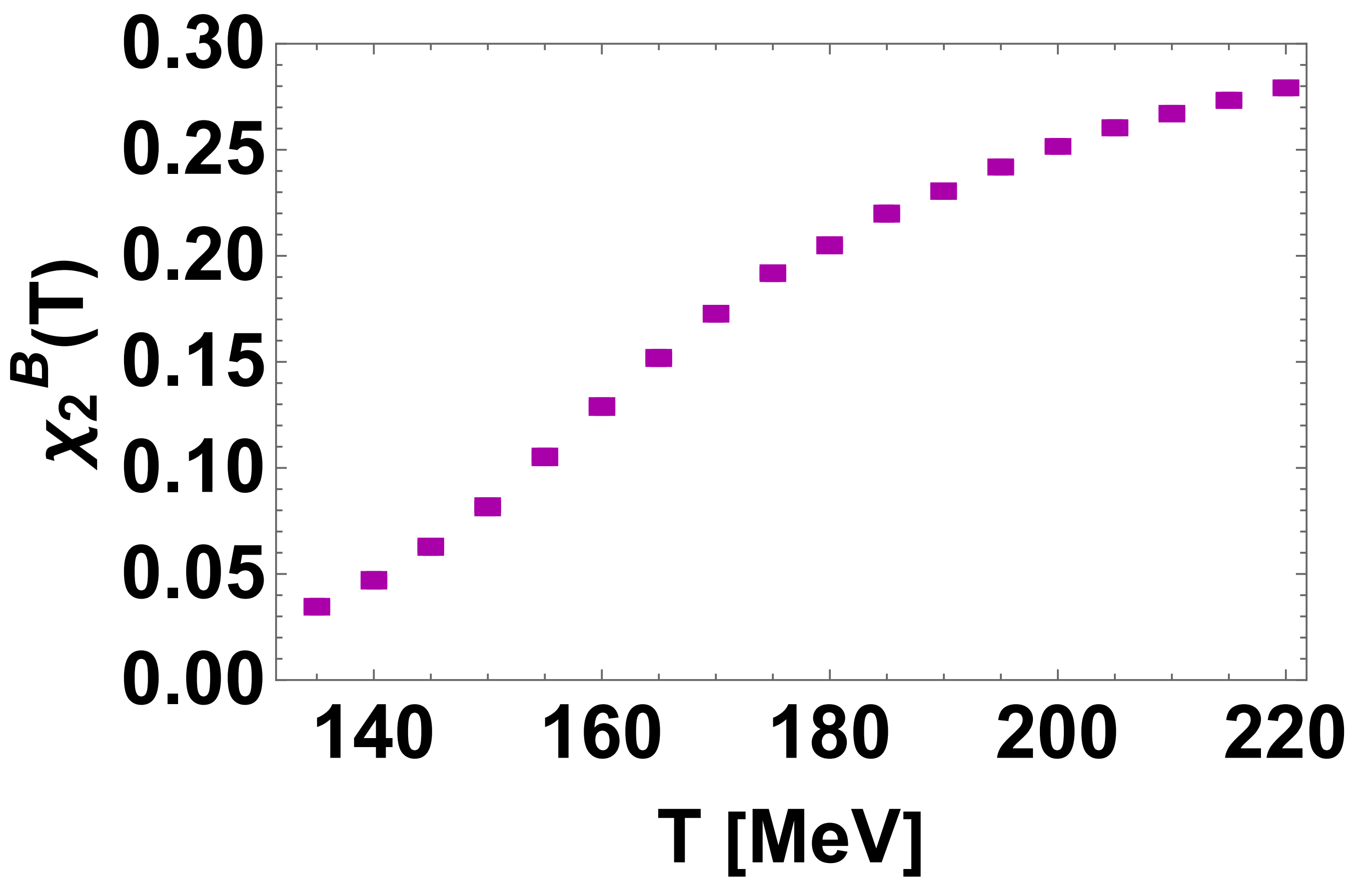

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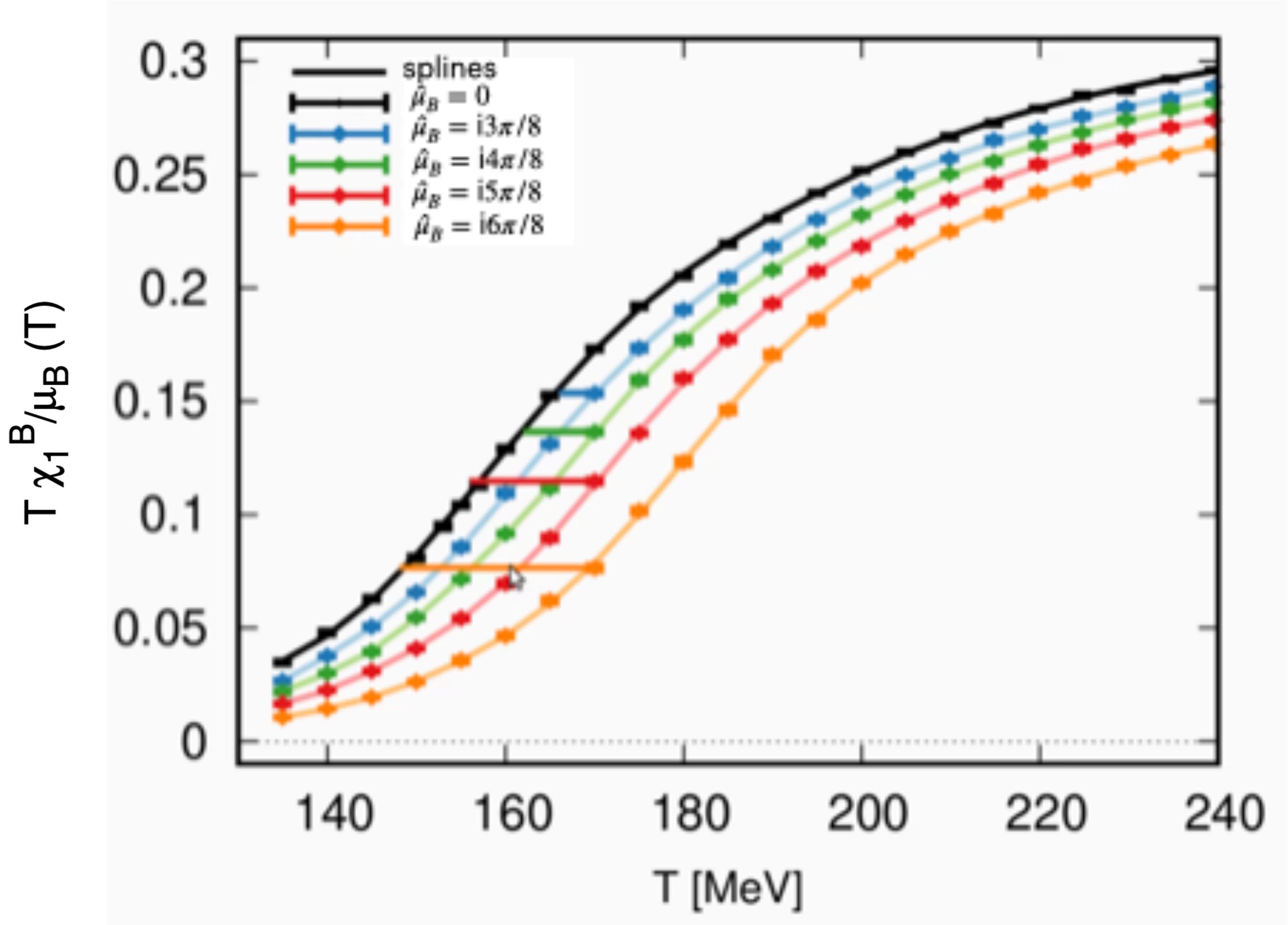

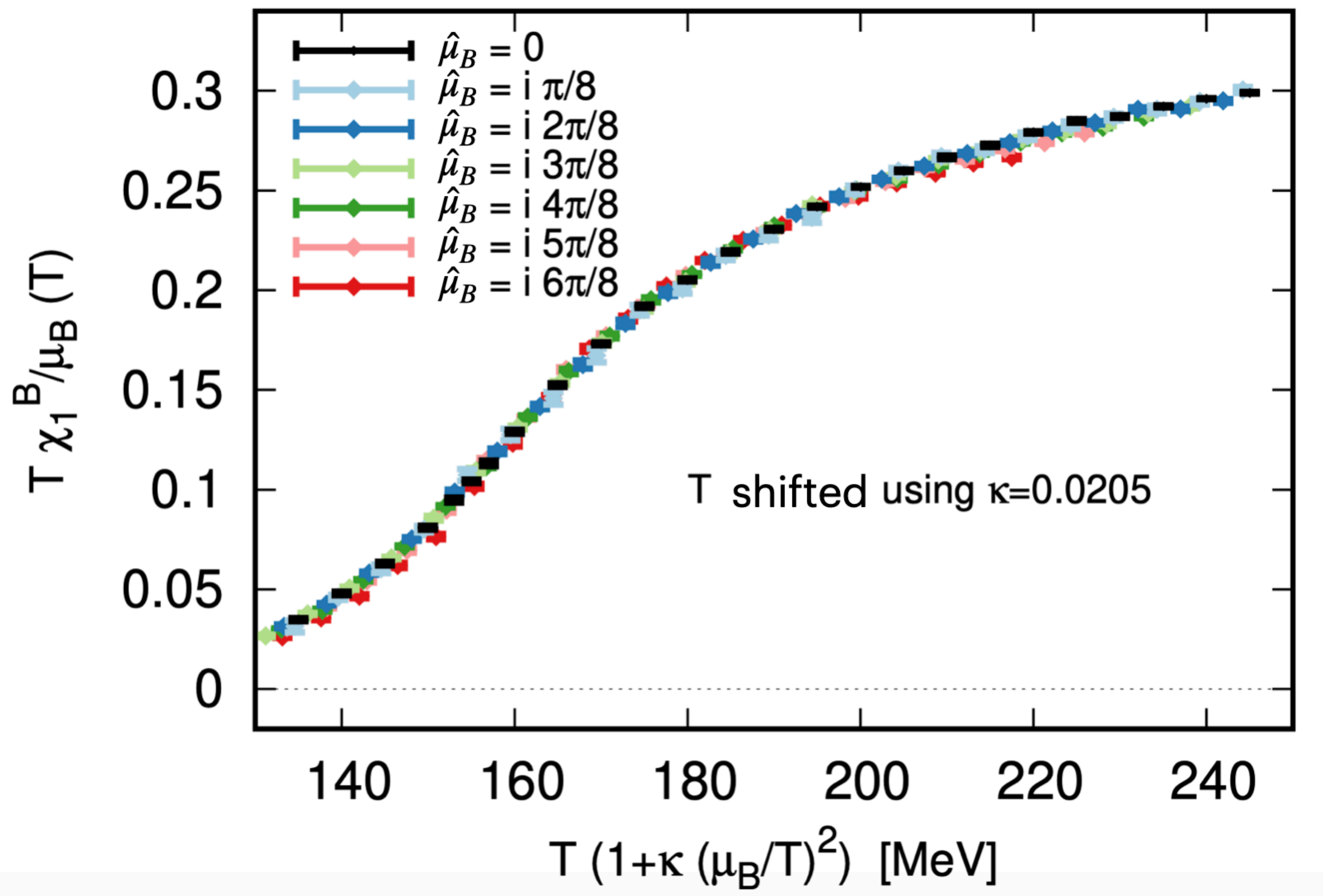

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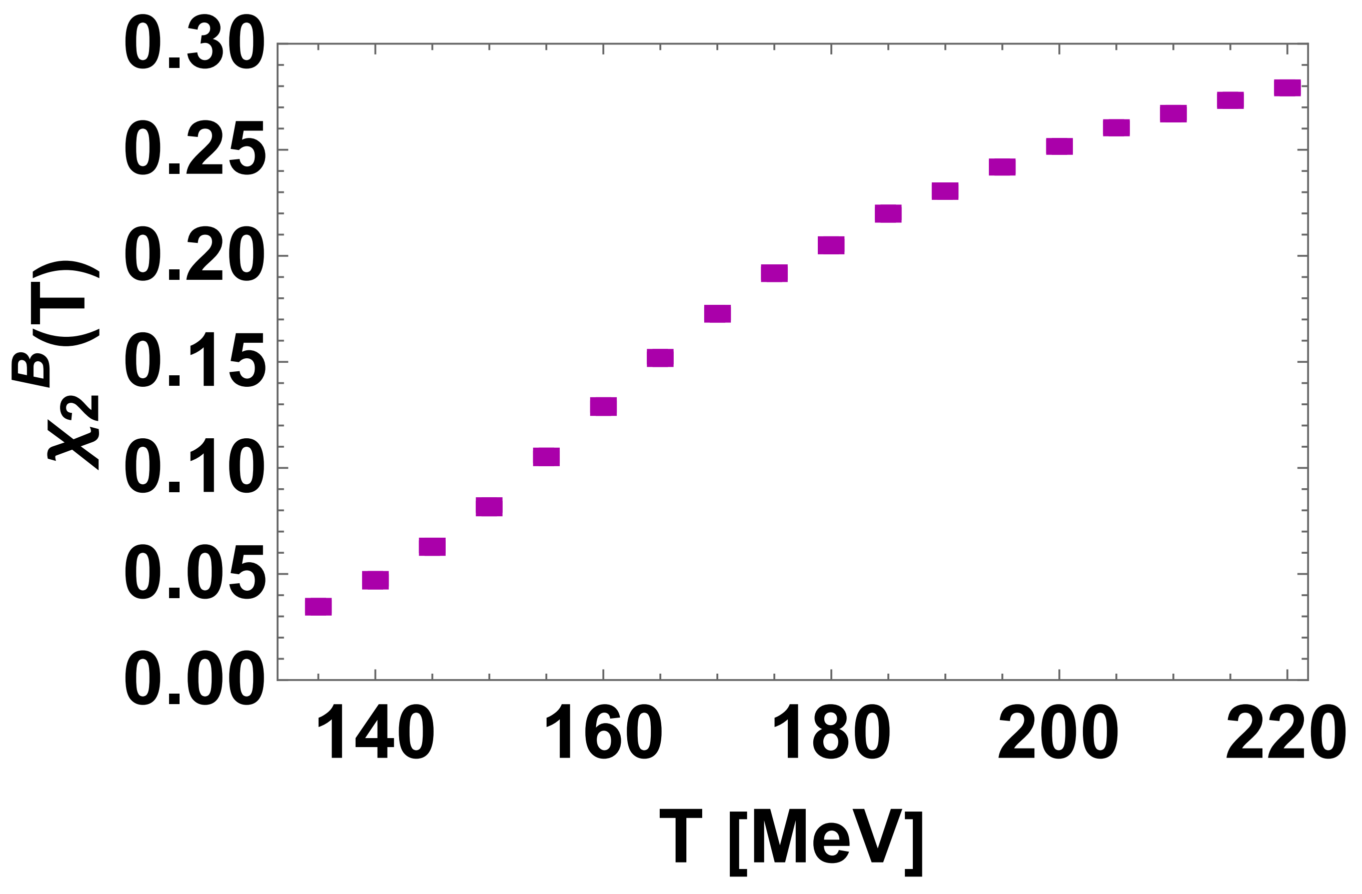

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$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

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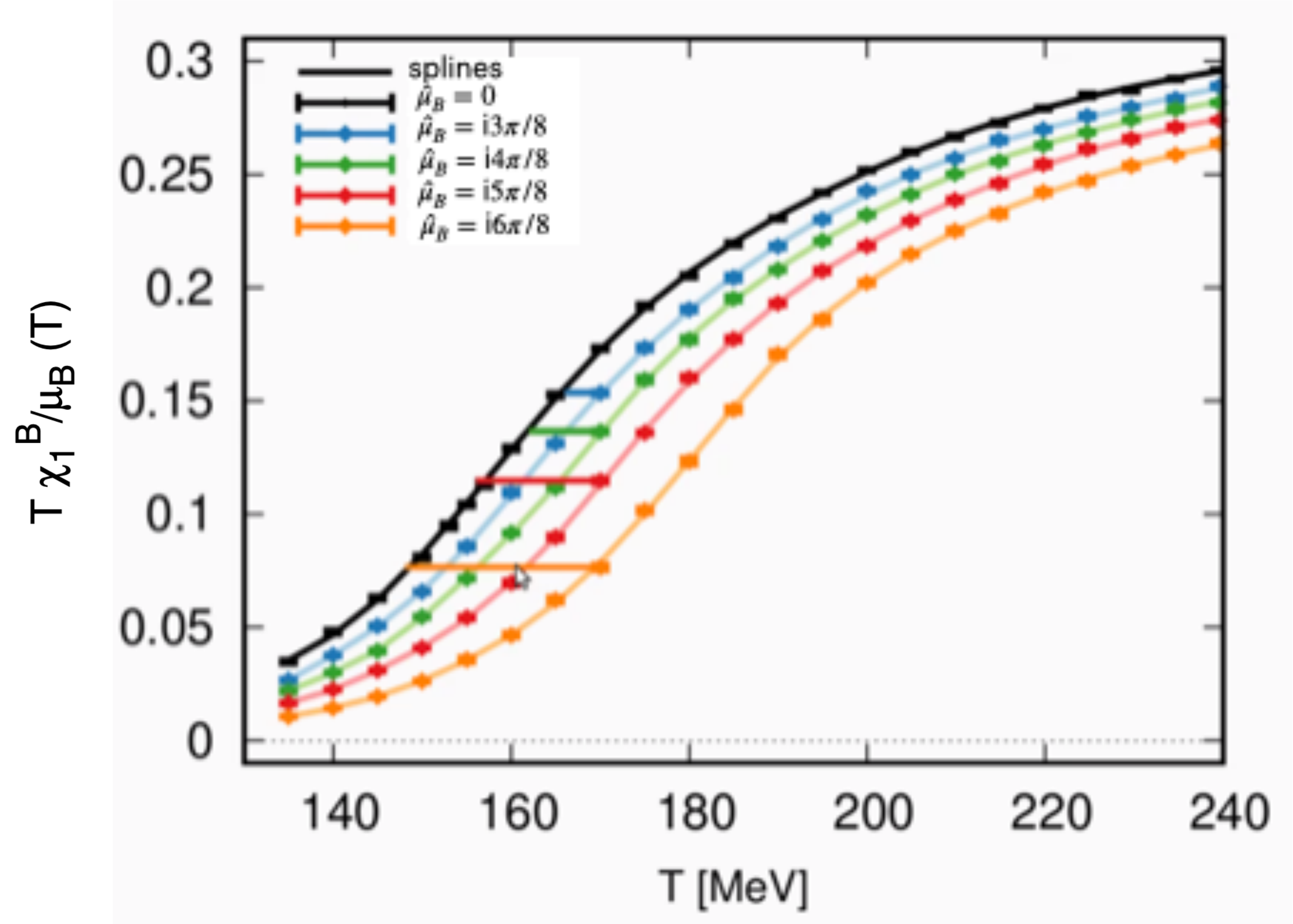

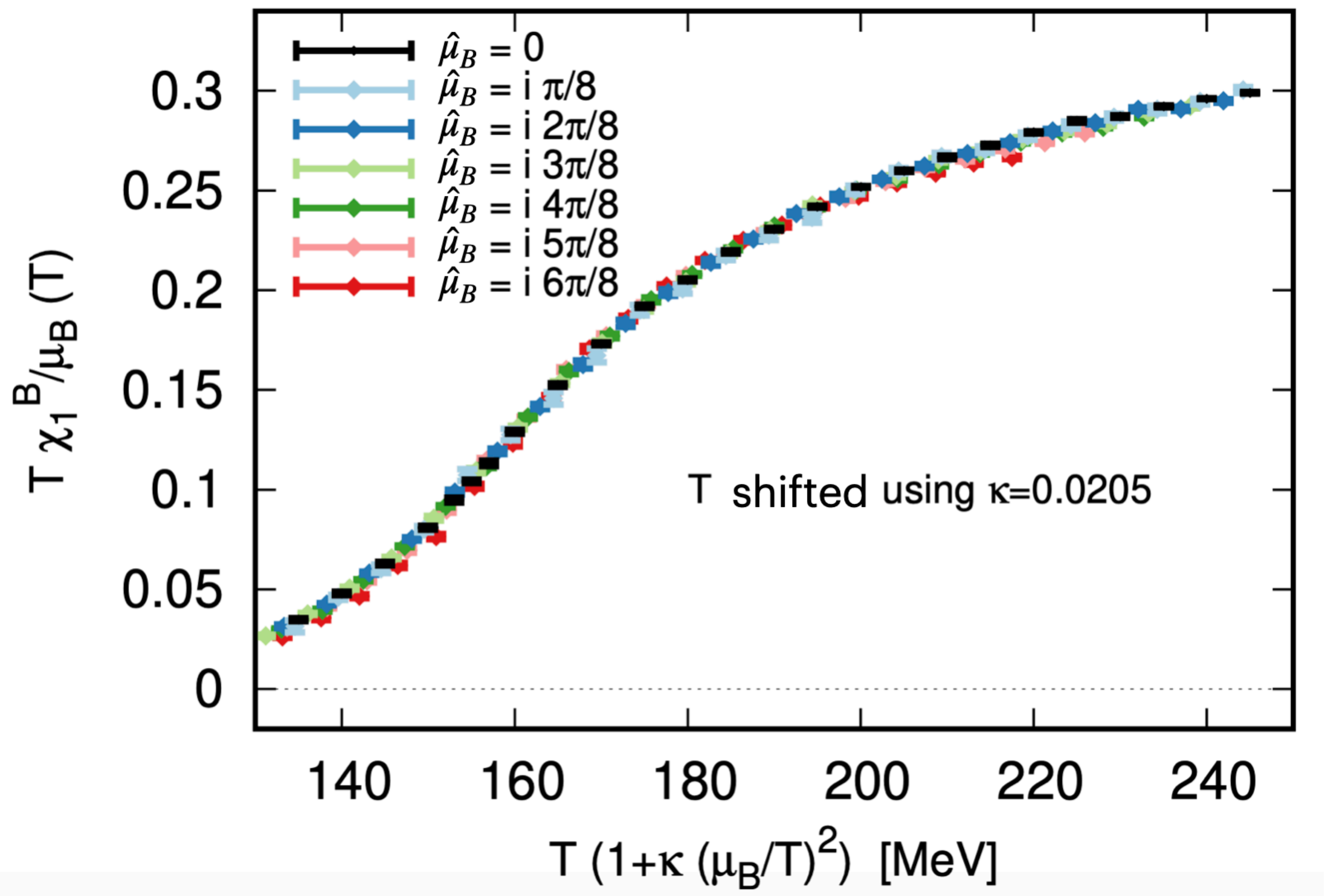

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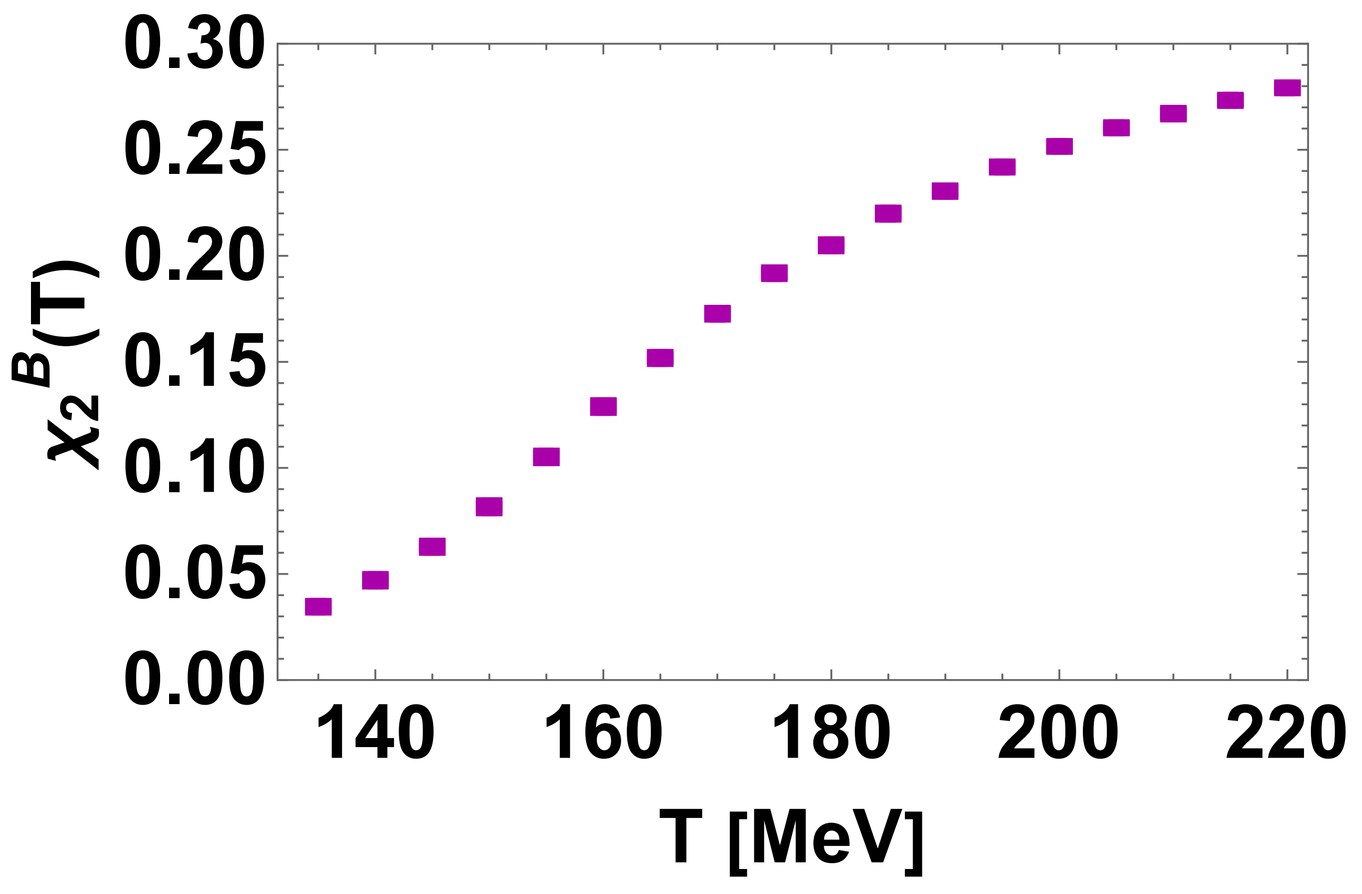

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- Uses few expansion terms
- μ_B dependence is captured in T-rescaling.
- Trusted up to $\frac{\mu_B}{T} = 3.5$ in the region where Critical point is expected

[M. K et al PhysRevD.109.094046]

Introducing Critical Point

Mapping 3D Ising to QCD

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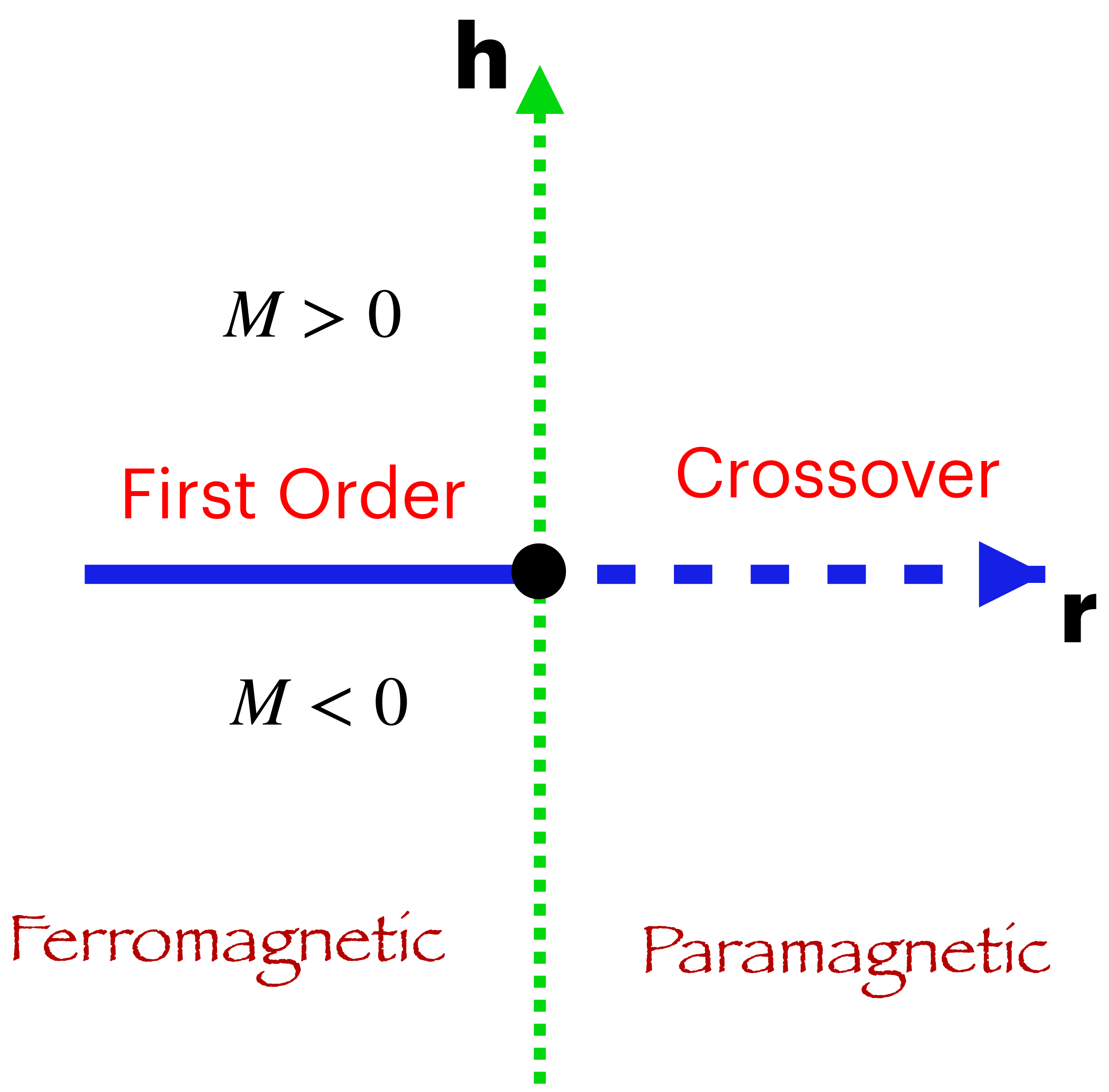

3D Ising coordinates

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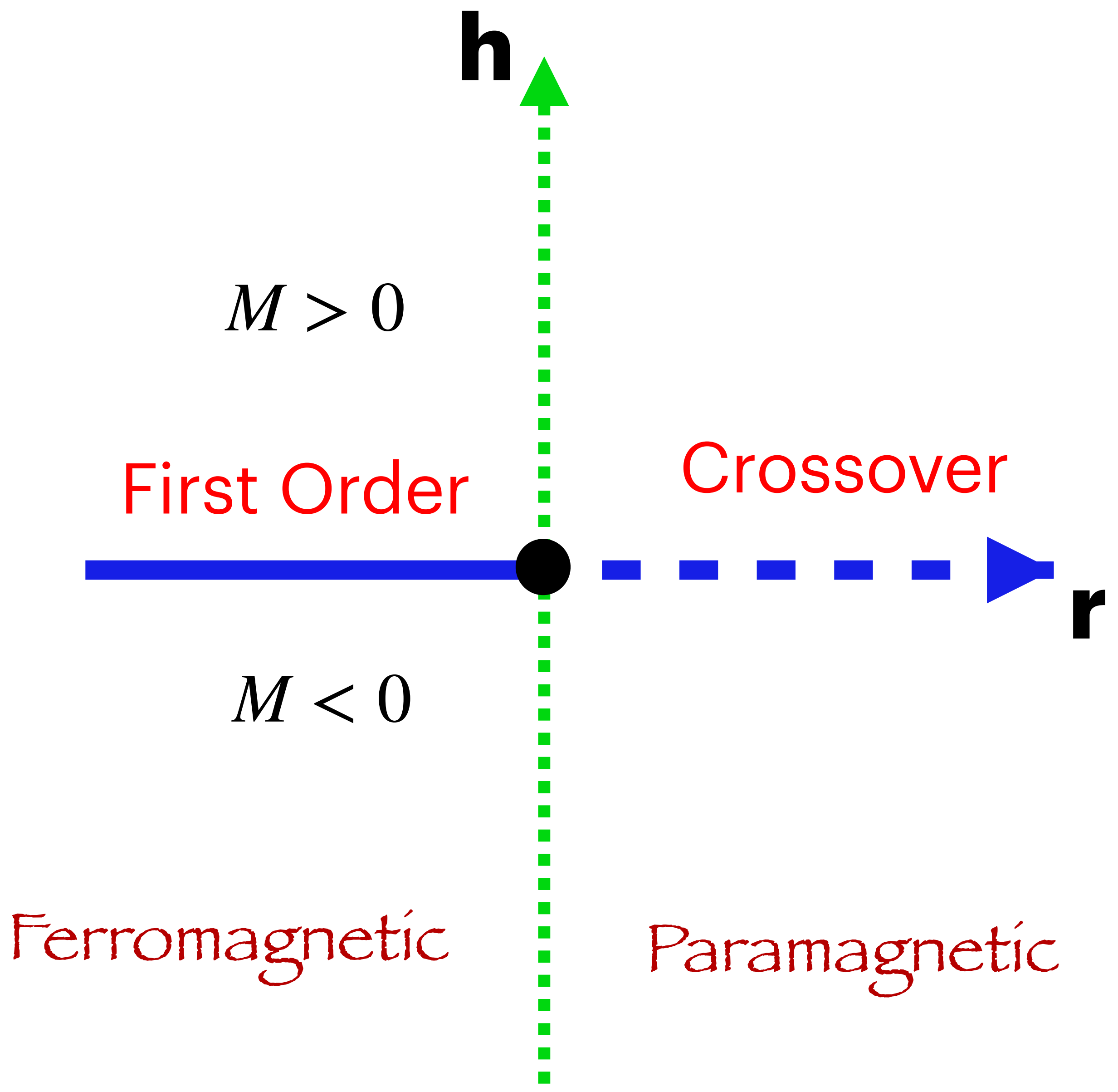
Mapping 3D Ising to QCD



3D Ising coordinates

$$\frac{T' - T_0}{T_C T'_{,T}} = -w' h \sin \alpha_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w'(-r\rho' - h \cos \alpha'_{12})$$

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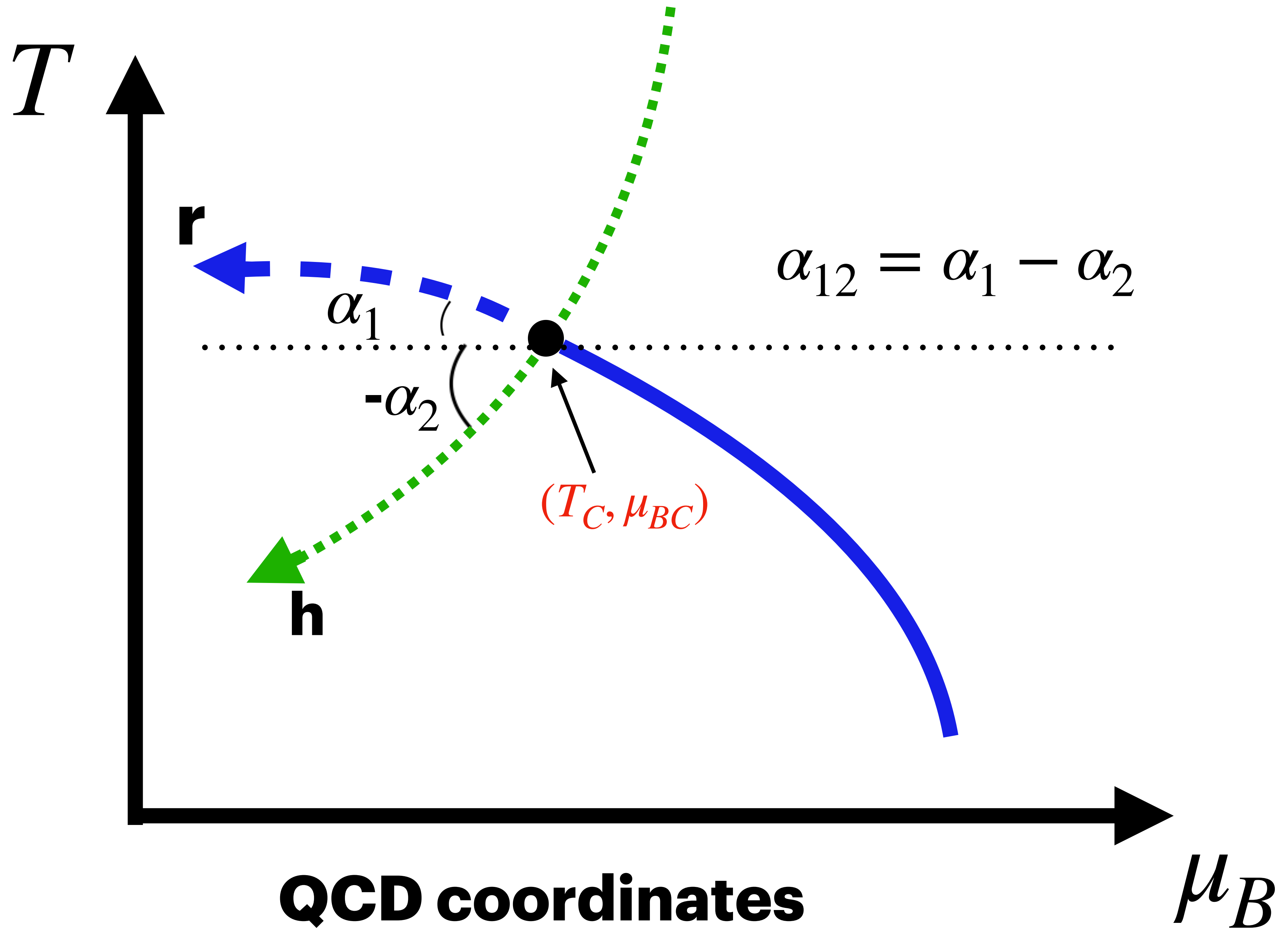

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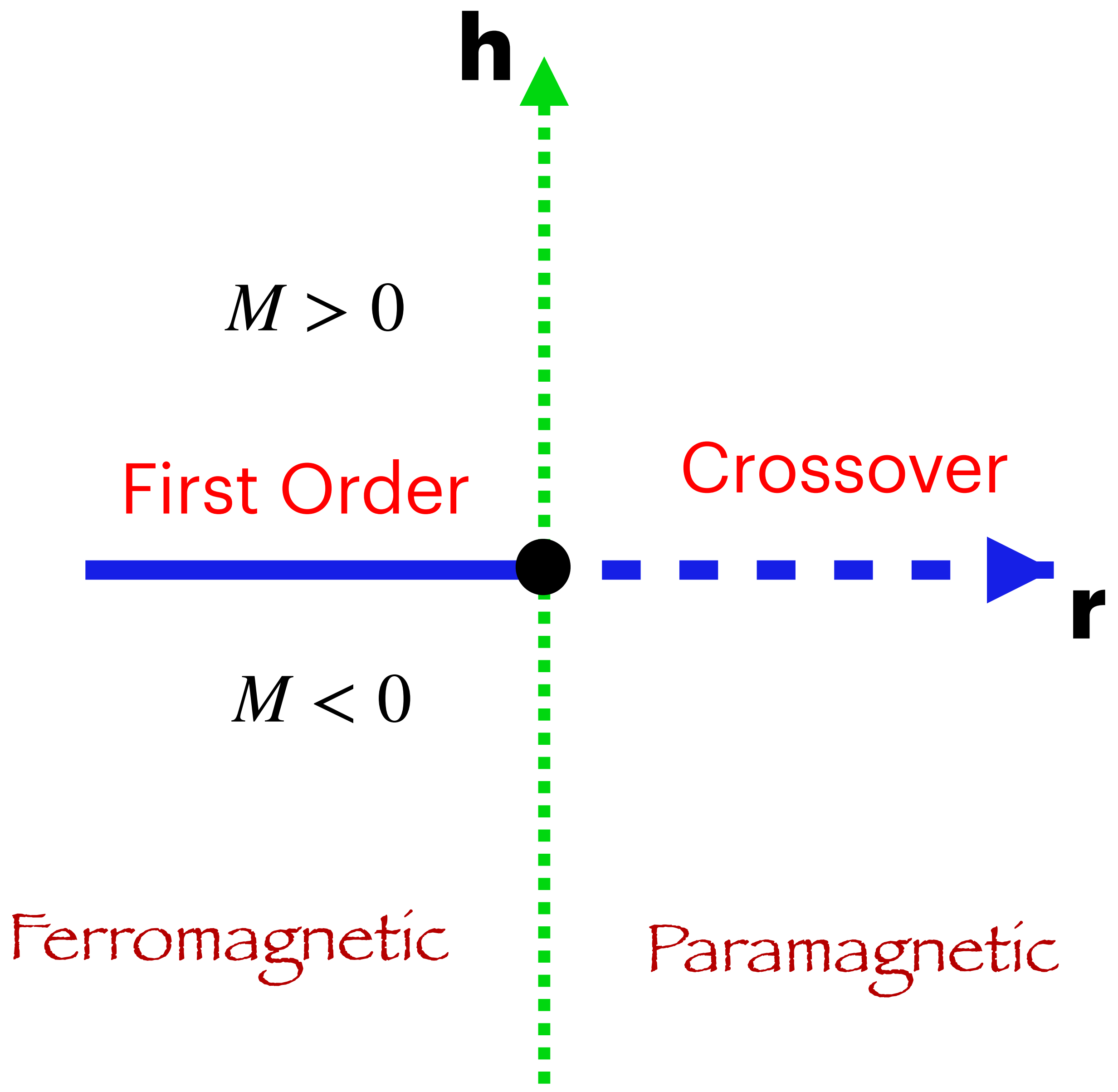

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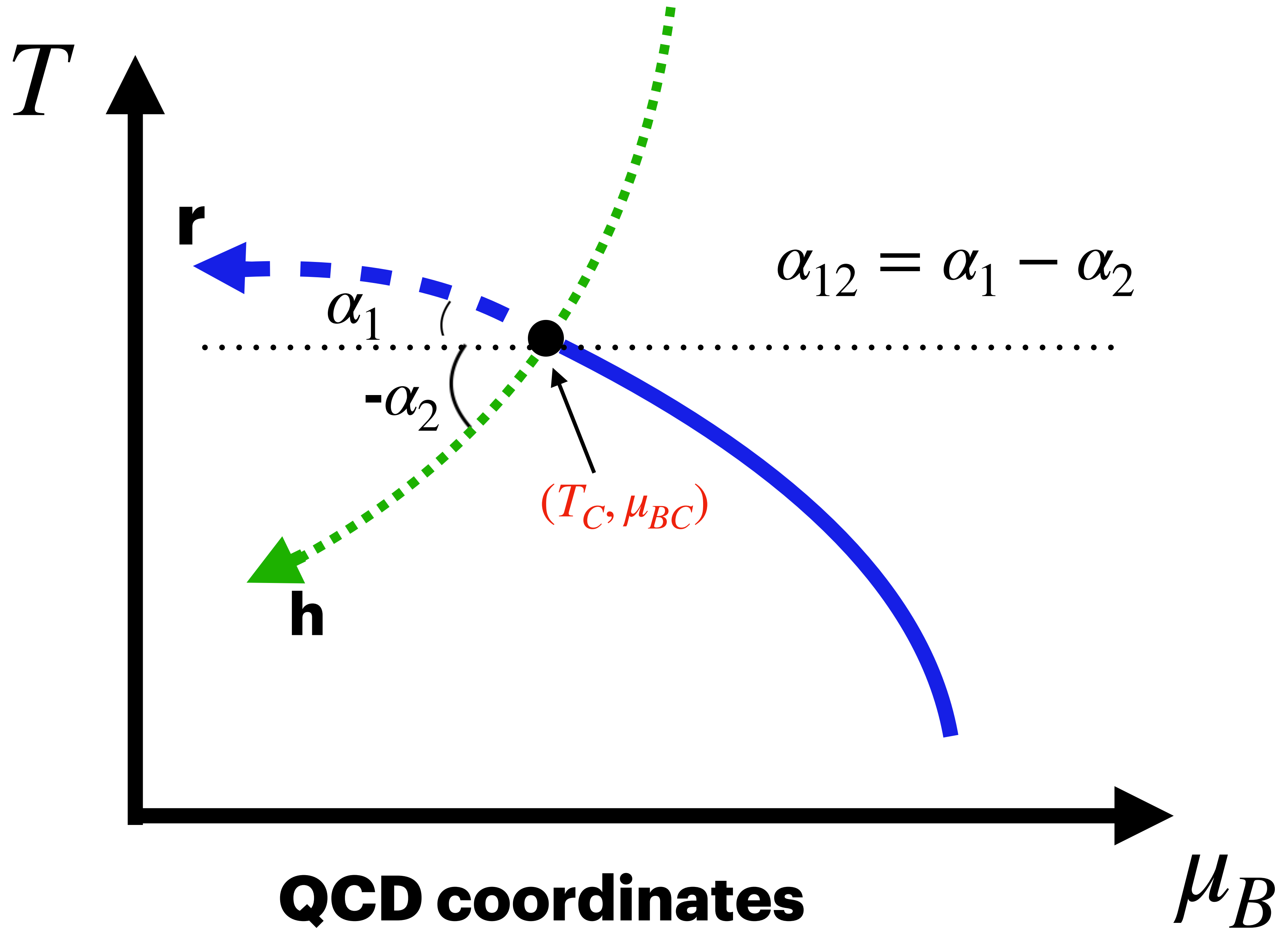

QCD coordinates

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- Free parameters μ_{BC}, T_C, w', ρ', α'_{12} can be fixed by the current physics knowledge

Merging Ising with Lattice (Ising-T ExS)

Full Baryon Density

$$\frac{n_B(T,\mu_B)}{T^3} = \chi_1^B(T,\mu_B) = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

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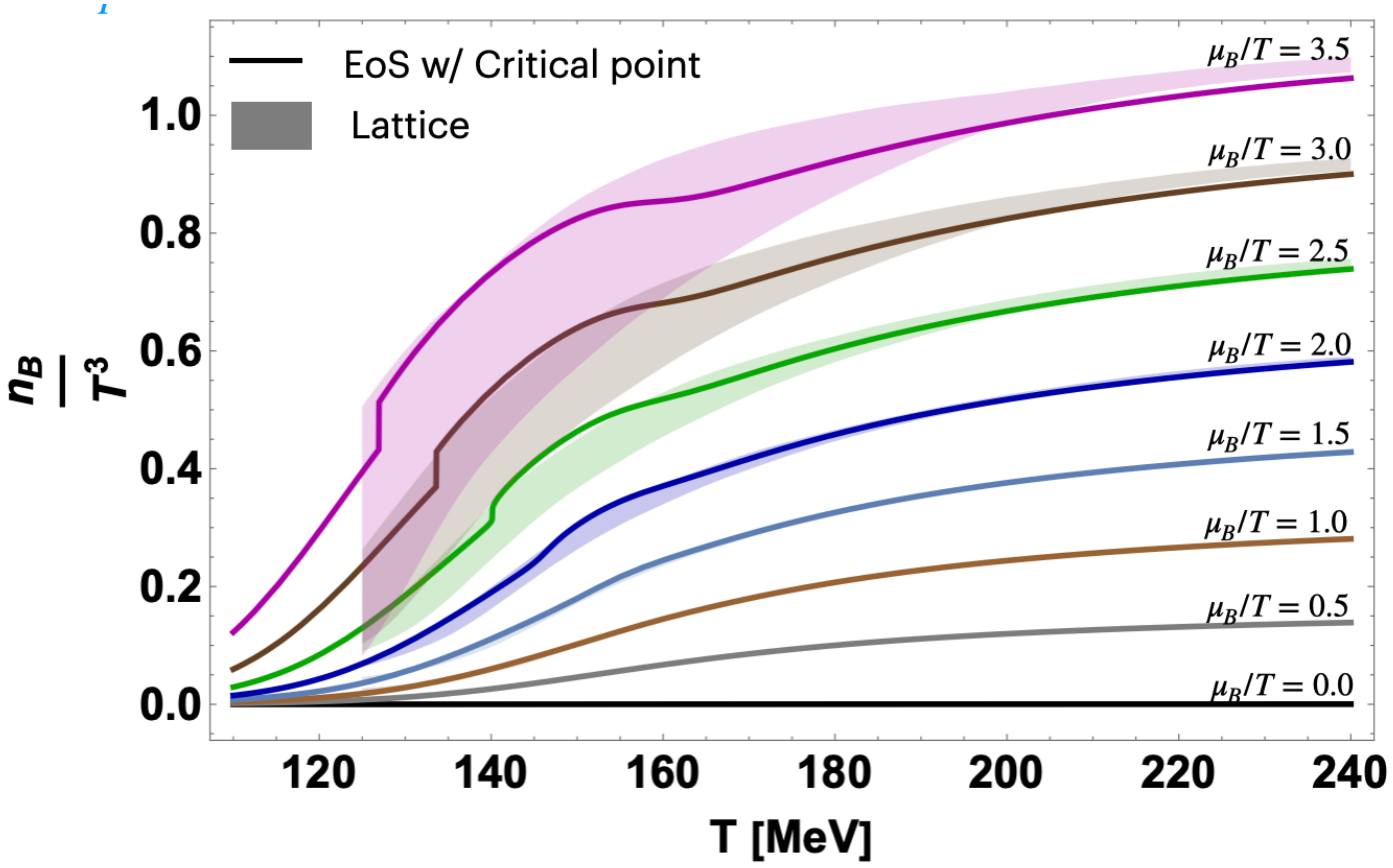


[M. K et al PhysRevD.109.094046]

Thermodynamic Observables

Parameter choice

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

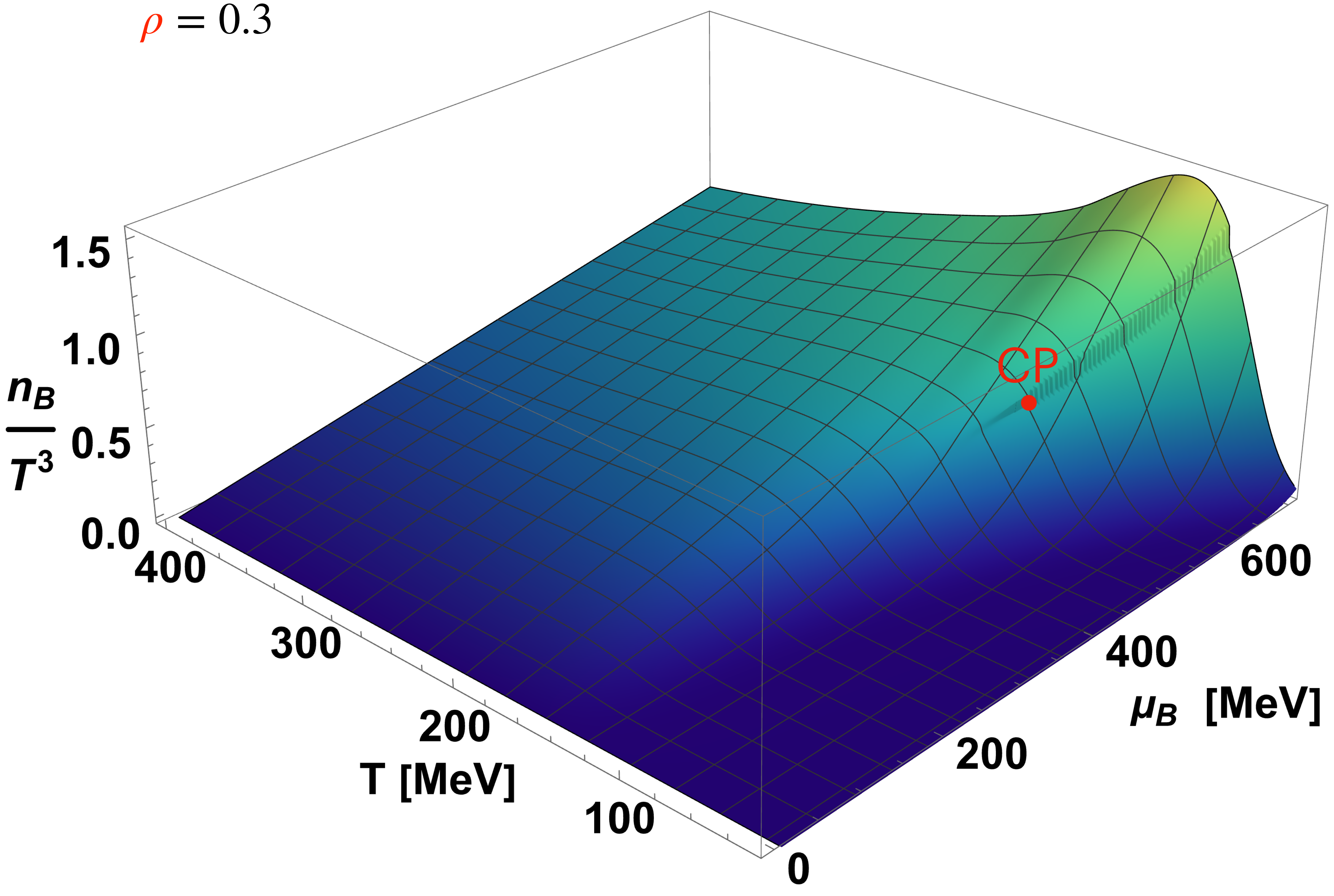
$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

$w = 15$

$\rho = 0.3$

Baryon Density



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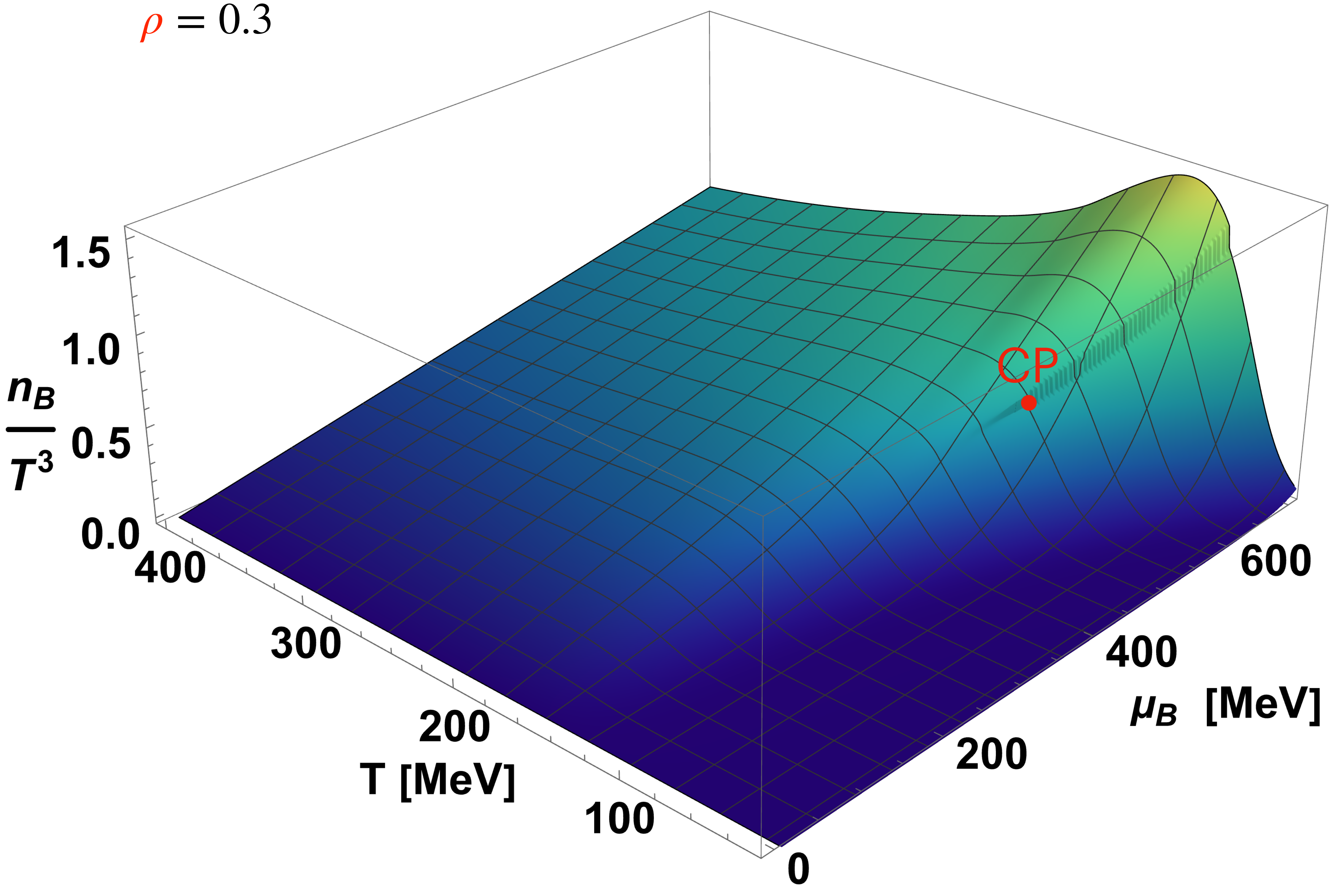
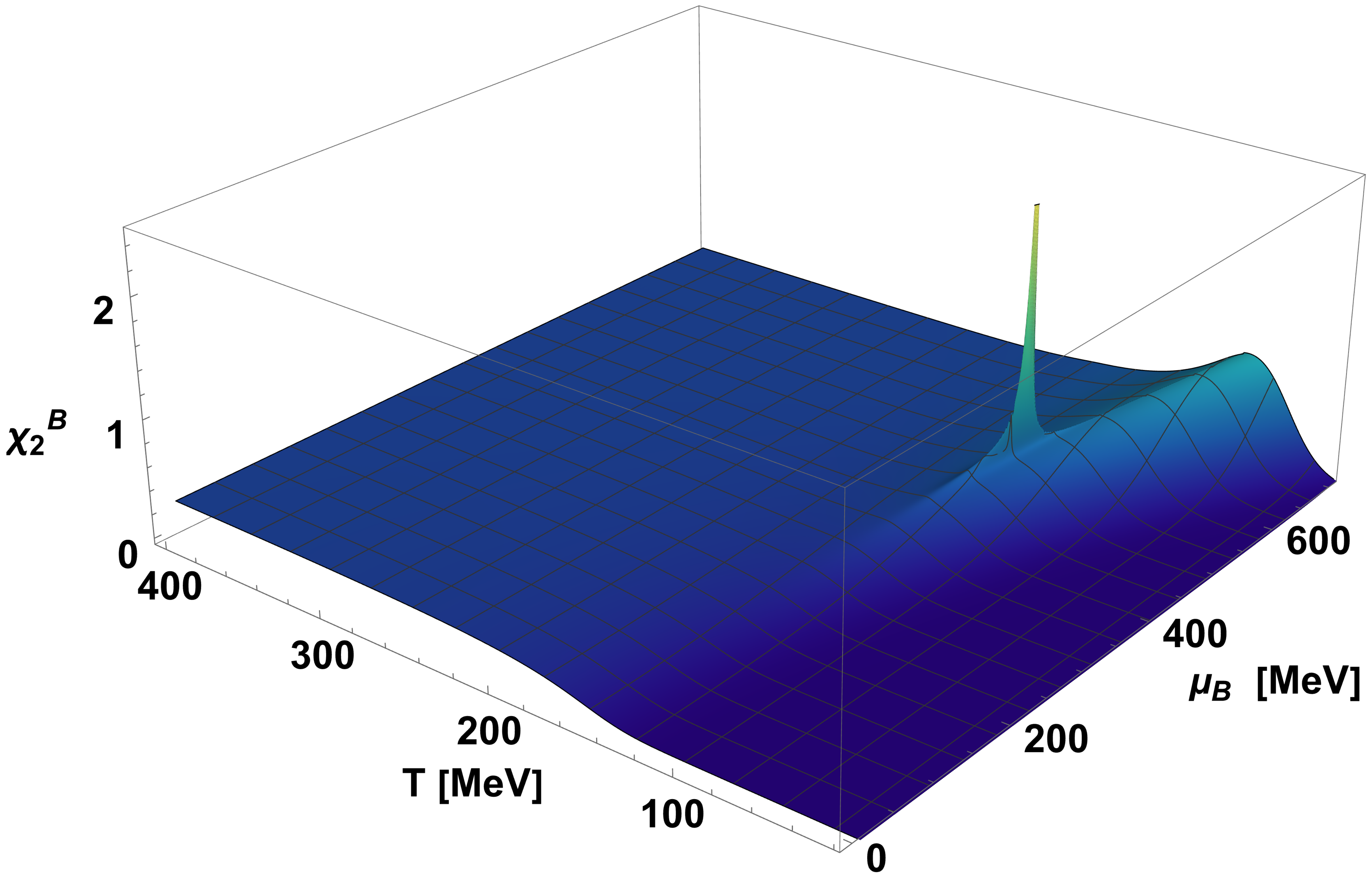


Baryon number susceptibility



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Summary and Conclusions

- We provide an Equation of State with enhanced coverage with 3D-Ising model Critical Point

DOI 10.5281/zenodo.10652326 (Open Software)

To appear in MUSES collaboration cyberinfrasture

- Our equation of state, has adjustable parameters, and can be used as input in **hydrodynamical simulations** to compare with experimental searches for the **critical point** in Beam Energy Scan II

Disclaimer! : We don’t predict the location of the critical point

Collaborators:

[M K, Steffen A Bass, Elena Bratkovskaya, Johannes Jahan, Pierre Moreau, Paolo Parotto, Damien Price, Claudia Ratti, Olga Soloveva, Mikhail Stephanov, Irene Gonzalez, Jorge A Muñoz, Volodymyr Vovchenko]