3D Ising Critical Point Mapped onto Lattice-based QCD Equation of State

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University of Houston

CPOD 2024 Berkeley, USA

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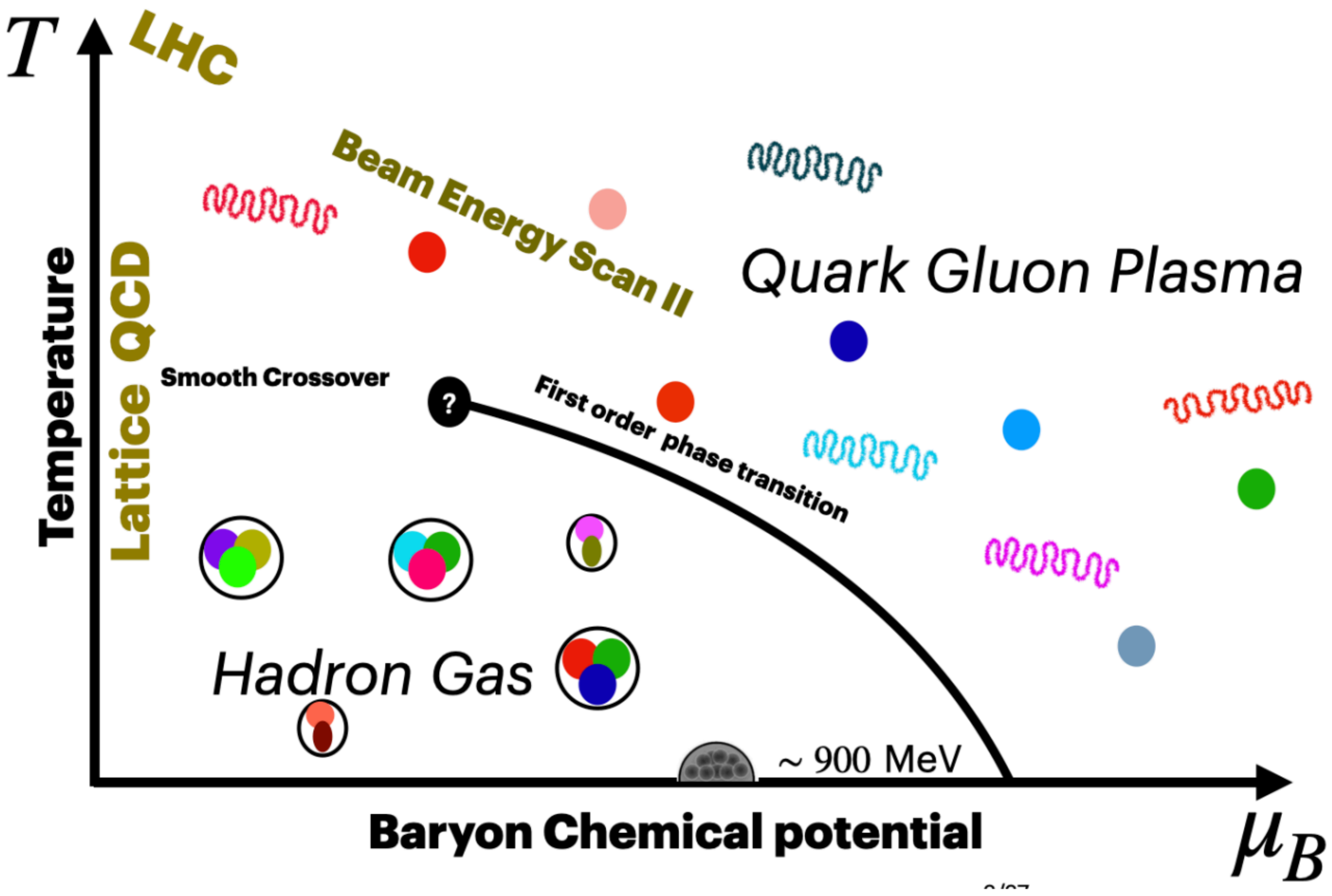


What we know

- At $\mu_B = 0$, de-confinement transition is well established (**Smooth crossover**)
- At finite μ_B, QCD **critical point** is expected but not yet seen
- Lattice simulations are challenging at Finite density **(Fermi-sign problem)**

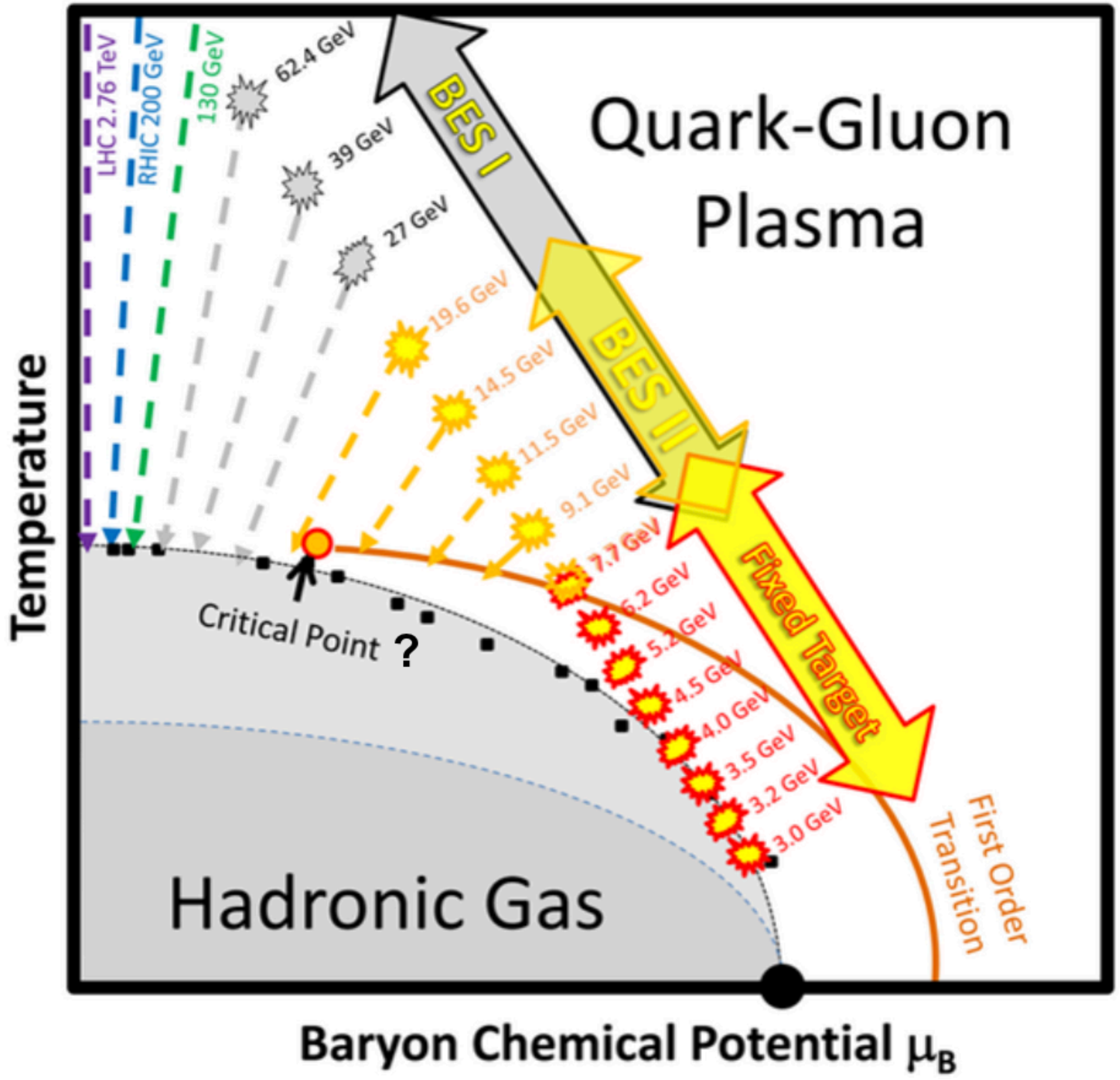
Attempts

- Direct simulation at finite μ_B like **re-weighting** are employed but limited to small volumes lattice

[Giordano, M. et al JHEP.05 088(2020)]
[Borsanyi, S. et al PhysRevD.105 014026(2022)]
[Borsanyi, S et al PhysRevD.107, L091503 (2023)]

- **Extrapolation schemes** are needed to describe finite density physics.

QCD Phase Diagram



Experiments

- Finite density physics is achieved by lowering the $\sqrt{S_{NN}}$ in **BES II program**

Theoretical interpretation

- **Hydrodynamic simulations** describe the evolution of the fireball in heavy ion collisions and neutron star Mergers
- An **Equation of state (EoS)** is required as an Input

It is crucial that the EoS used encompasses all existing physics knowledge with adjustable parameters

Part 1: Taylor Expansion

Taylor: Lattice QCD results

Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!}\chi_{2n}(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^{2n}$$

[Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]
[Bazavov, A et al PhysRevD.95, 054504 (2017)]

$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\Bigg|_{\mu_B=0}$$

[Borsanyi, S. et al JHEP 10 205 (2018)]
[Bazavov, A et al PhysRevD.95, 054504 (2017)]

Taylor: Lattice QCD results

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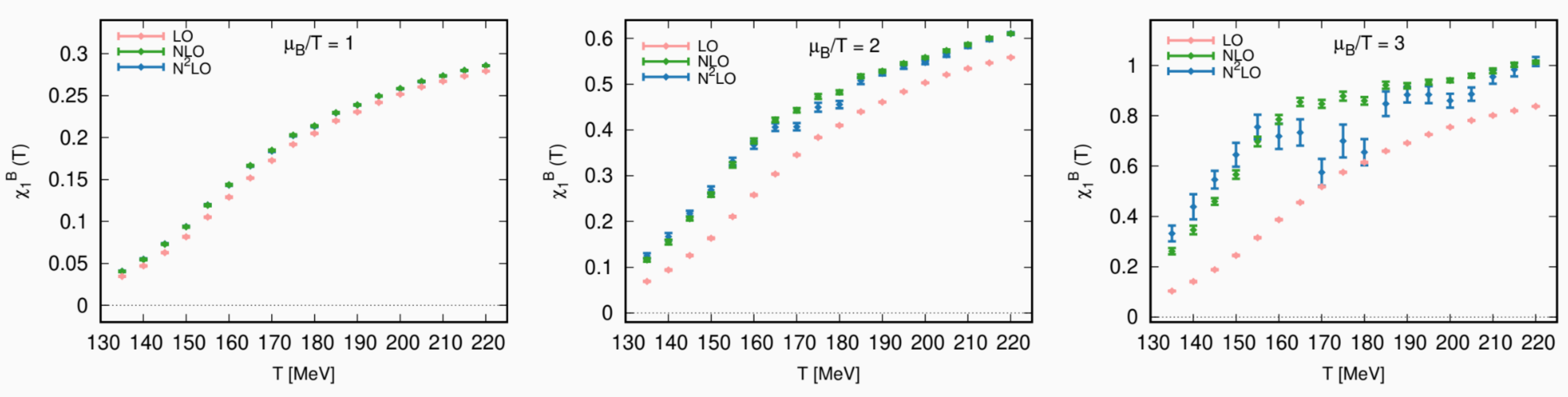
$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\bigg|_{\mu_B=0}$$

Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational effort**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]

[Borsanyi , S et al arXiV:2312.07528v1. (2023)]



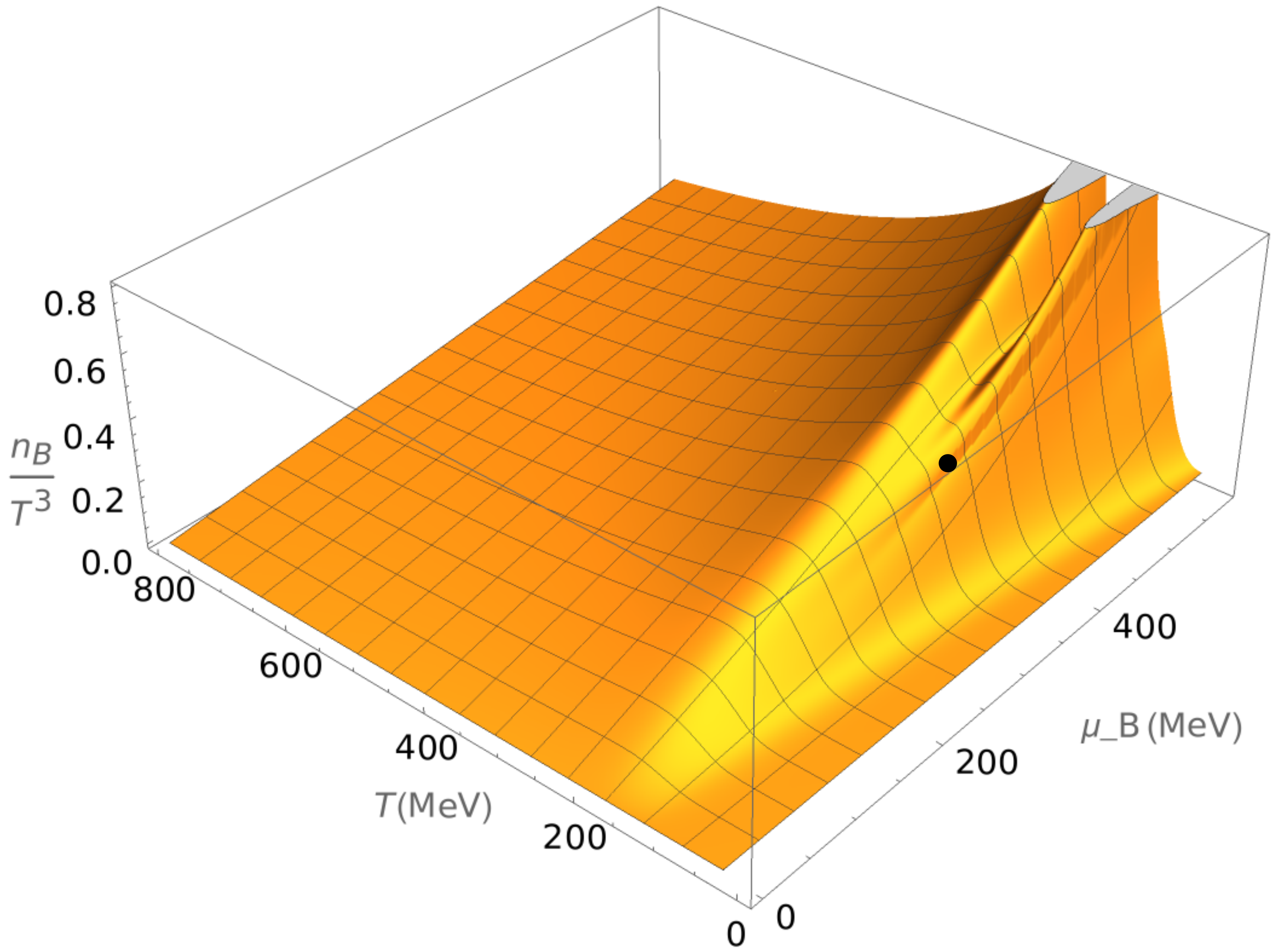
[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]

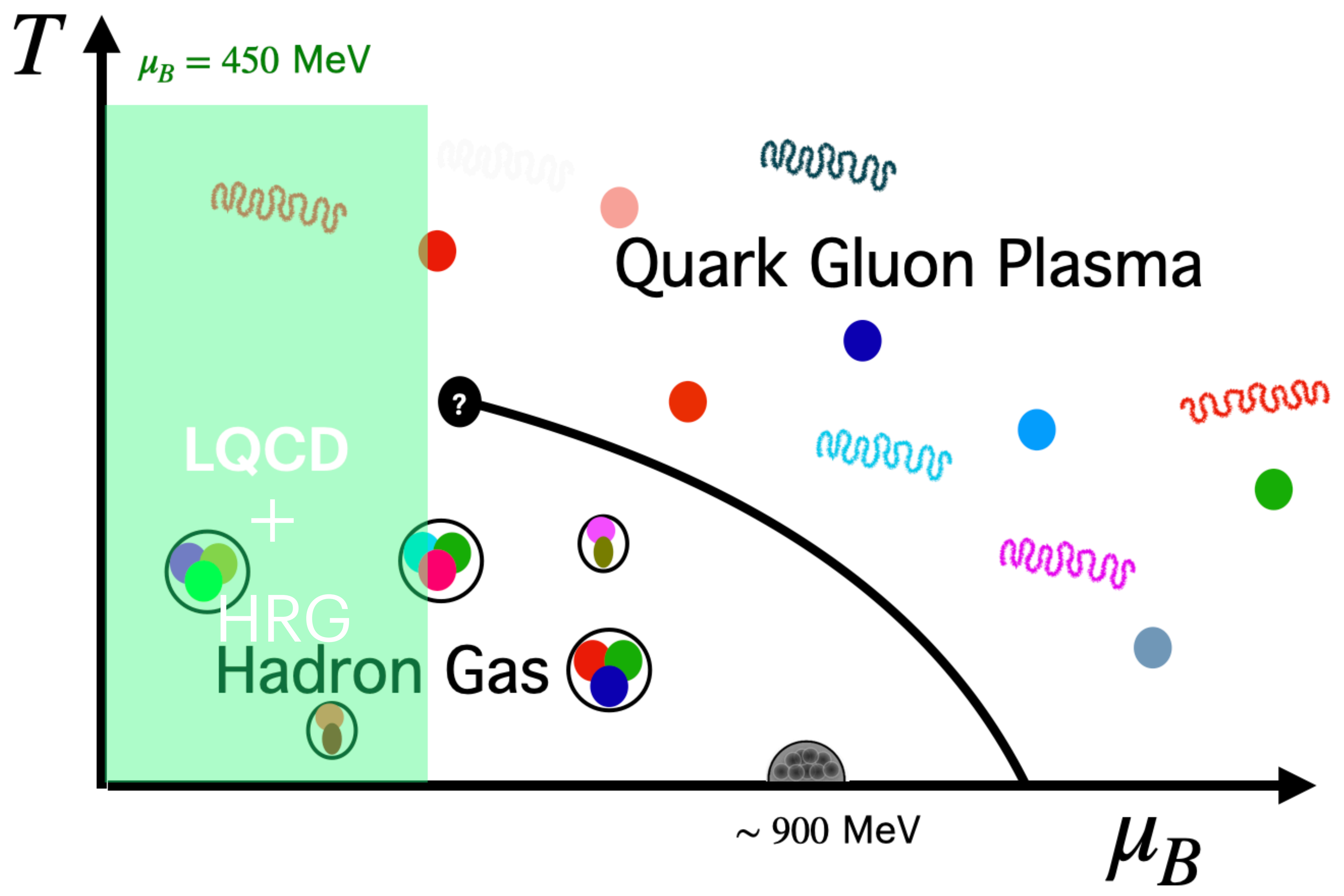
Taylor: merging of lattice QCD results and critical behavior

$$n_B(T,\mu_B) = T^3 \sum_{n=0}^{2} \frac{1}{(2n-1)!} \chi_{2n}^{non-Ising}(T) \left(\frac{\mu_B}{T}\right)^{2n-1} + \frac{T_C^4}{T} n_B^{Ising}(T,\mu_B)$$

Taylor expansion up to $\mathcal{O}((\mu_B/T)^4)$

$$\chi_n^{lat}(T) = \chi_n^{non-Ising}(T) + \frac{T_C^4}{T^4}\chi_n^{Ising}(T)$$



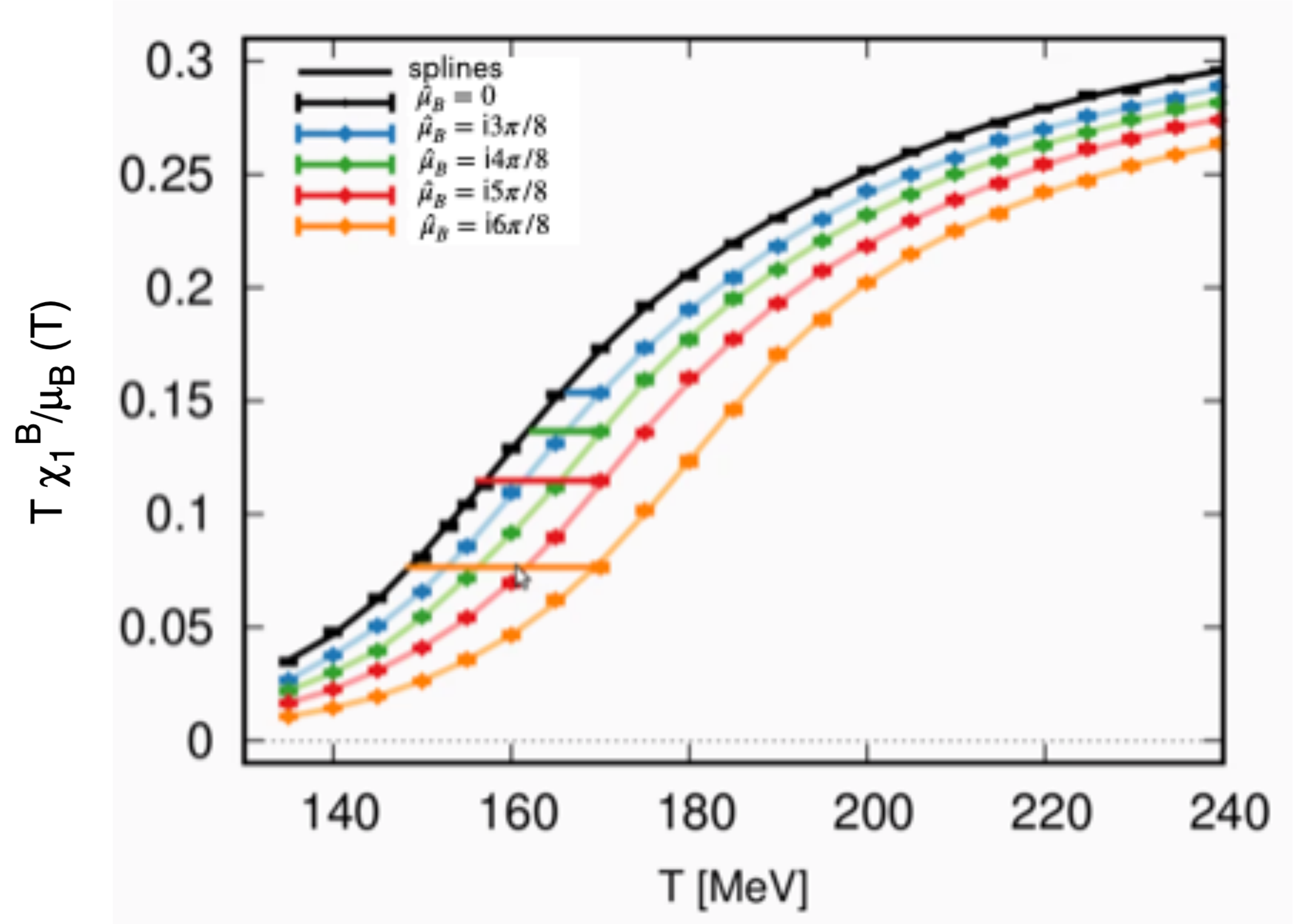


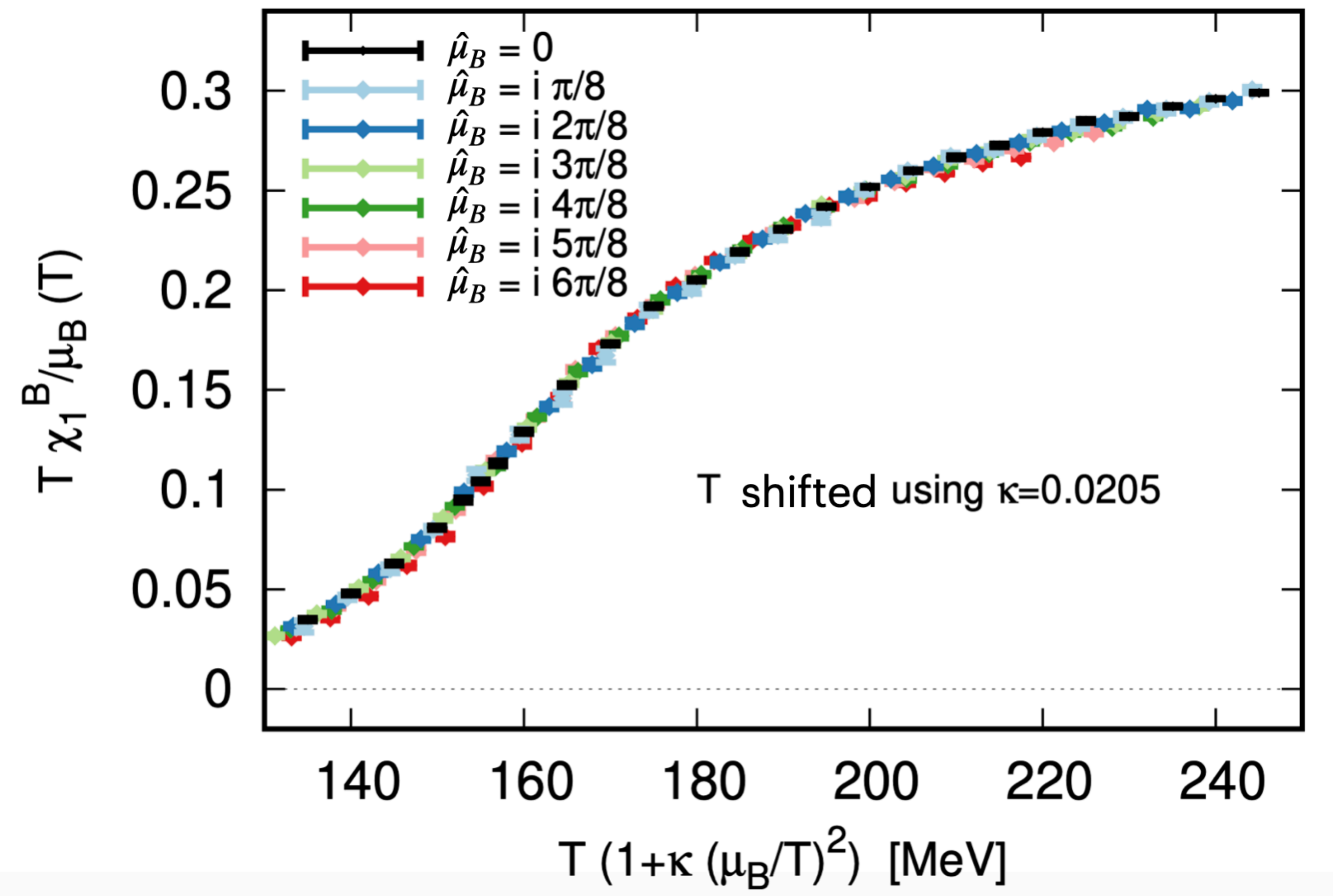
[Parotto, P et al PhysRevC. 108(1), 101.034901(2020)]
[Karthein, J, et al arXiv:2110.00622.(2021)]

Part 2: T’ Expansion Scheme (T ExS)

T′ Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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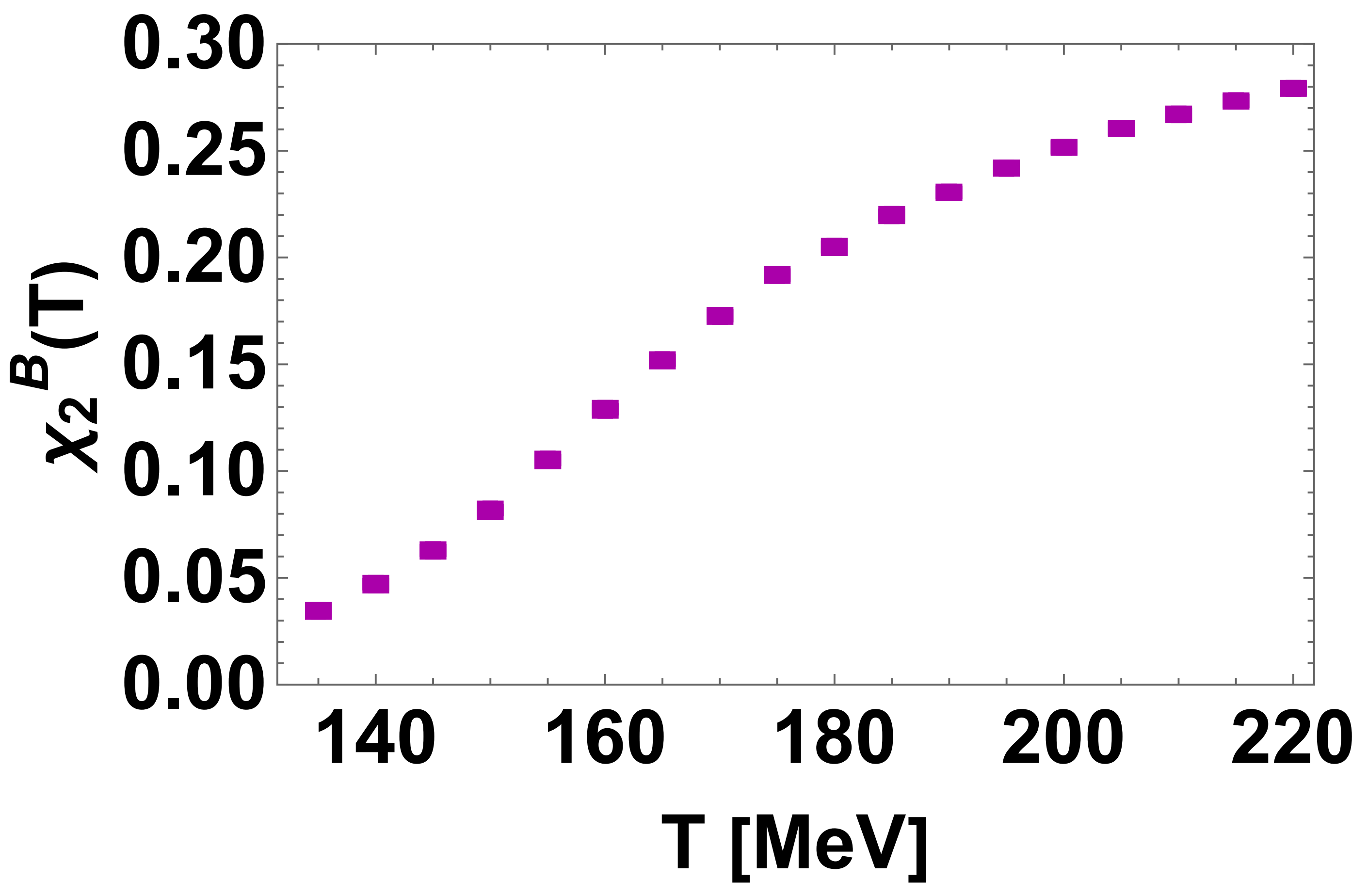


[Borsányi, S et al PhysRev.L. 108(1), 101.034901(2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

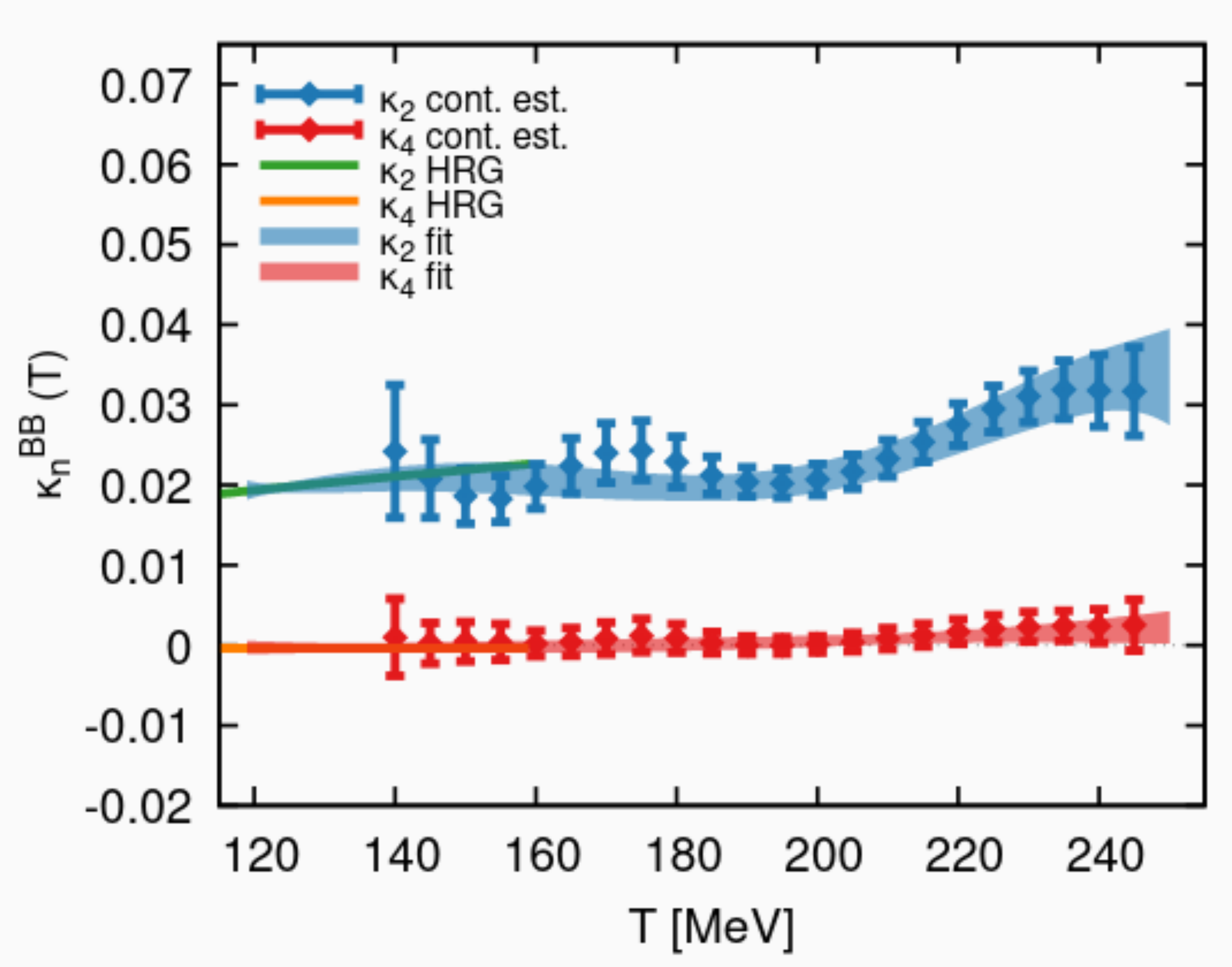
- Uses few expansion terms
- μ_B dependence is captured in T-rescaling.
- Trusted up to $\frac{\mu_B}{T} = 3.5$

T′ Expansion scheme (T ExS)

Relationship between **Taylor expansion** and **T′ expansion**

- $\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\partial\chi_2'^B(T)}$

- $\kappa_4^{BB}(T) = \frac{1}{360T\chi_2'^B(T)^3}\left(3\chi_2'^{B2}\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$



[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]

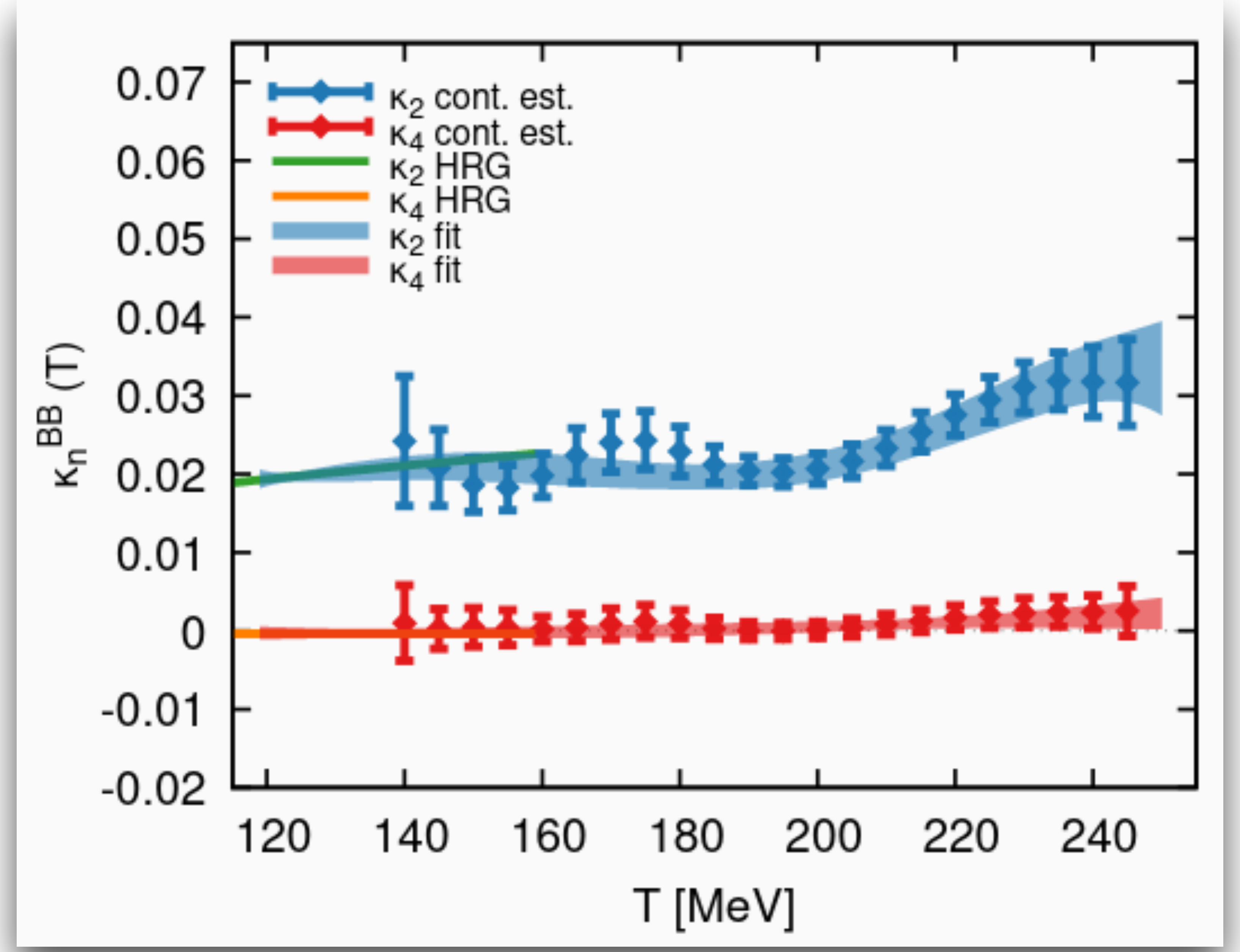
Pros

- $\kappa_2(T)$ **is fairly constant over a large T-Range**
- **There is a separation of scale between** $\kappa_2(T)$ **and** $\kappa_4(T)$
- $\kappa_4(T)$ **is almost zero** $\rightarrow$ **faster convergence**
- **A good agreement with HRG results at Low Temperature**

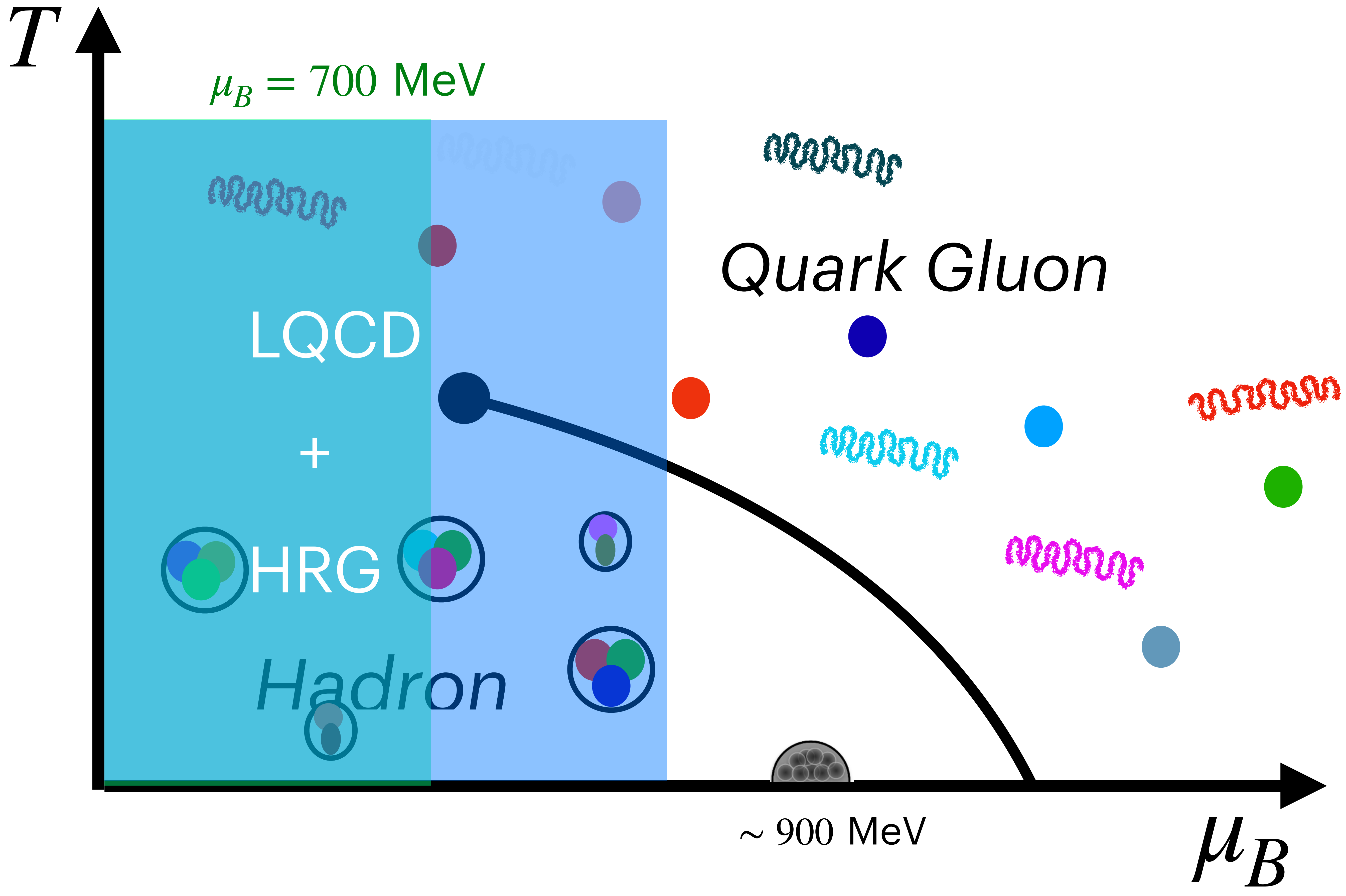
T′ Expansion scheme (T ExS)

Relationship between **Taylor expansion** and **T′ expansion**

- $\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\partial\chi_2^{\prime B}(T)}$
- $\kappa_4^{BB}(T) = \frac{1}{360T\chi_2^{\prime B}(T)^3}\left(3\chi_2^{\prime B^2}\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$



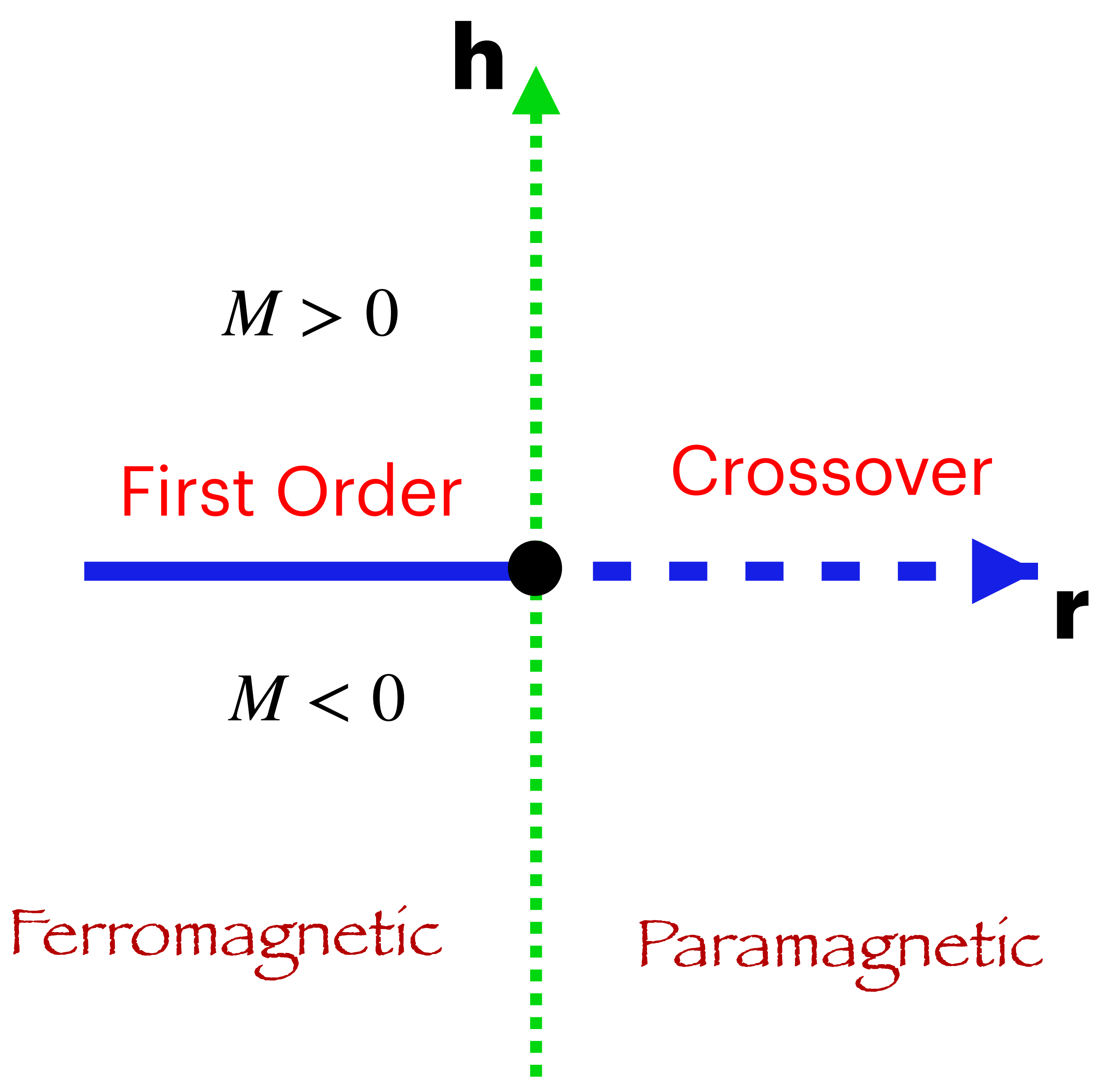
[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]



Part 3: Introducing Critical Point (3D-Ising)

Introducing Critical Point

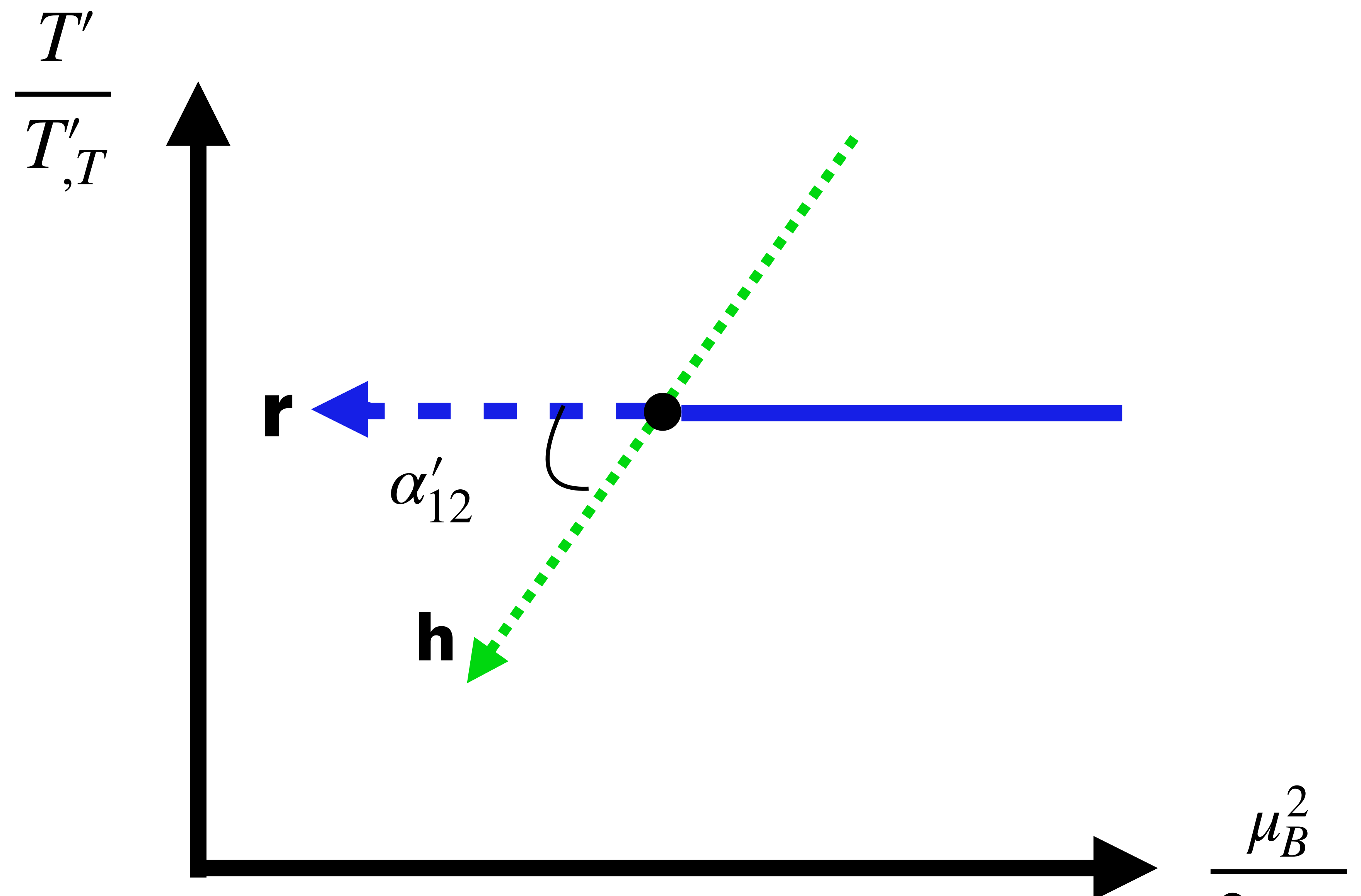
Mapping 3D Ising to QCD



3D Ising coordinates

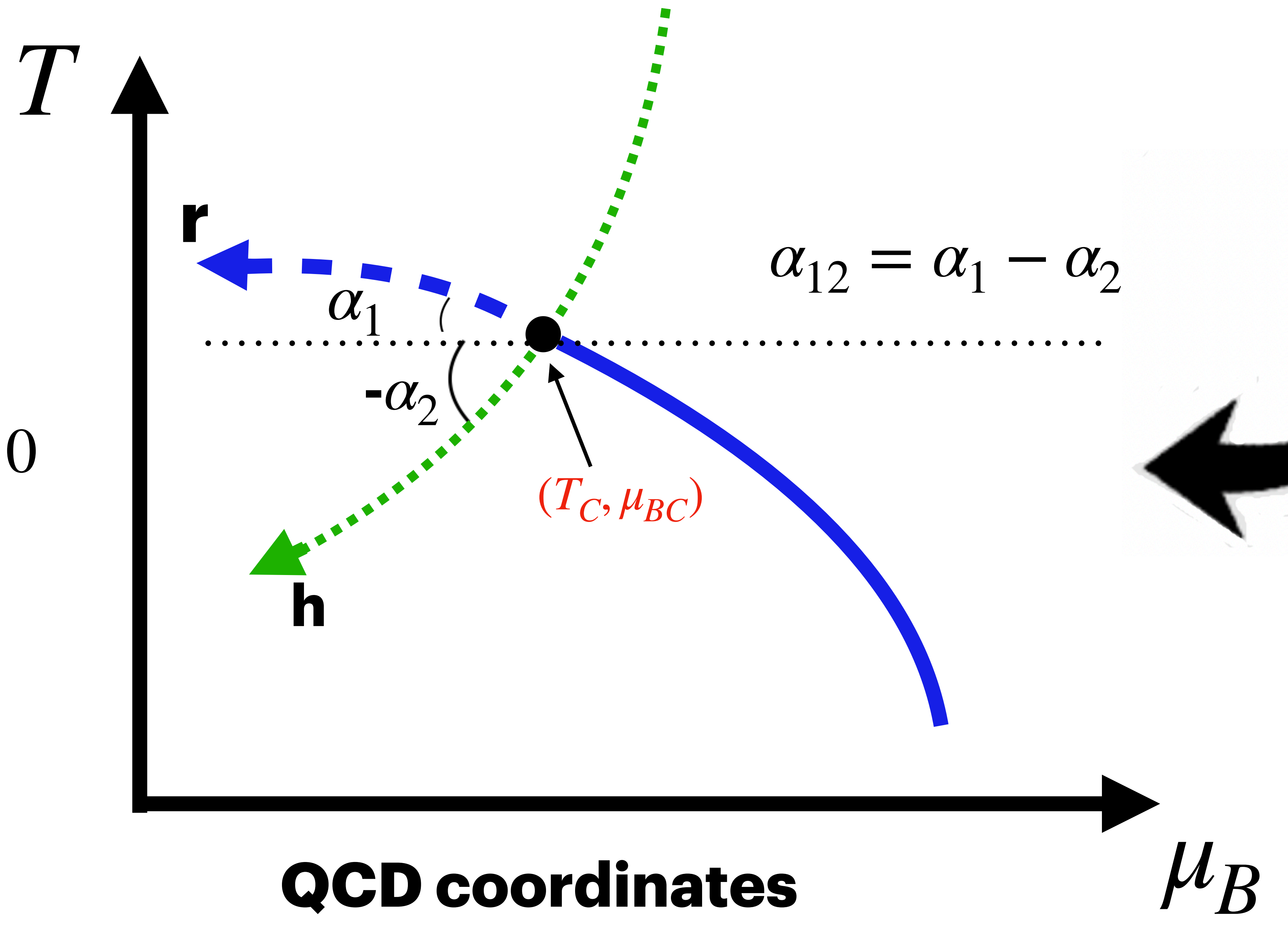
$$\frac{T' - T_0}{T_C T'_{,T}} = - w' h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w' (-r \rho' - h \cos \alpha'_{12})$$



T' expansion coordinates

$$T' = T \left[1 + \left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4 \right]$$



QCD coordinates

$T'_{,T} = (\partial T' / \partial T)_\mu$ at the critical point

T_0 - Transition temperature at $\mu_B = 0$

$\mu_{BC}, T_C, w', \rho', \alpha'_{12}$ - Free parameters

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Important relations

Relationship with BEST collaboration EoS

- The mapping is not universal
- Quadratic mapping is related to BEST Collaboration (linear) mapping

$$\mu_{BC}, T_C, \alpha'_{12}, w', \rho' \longrightarrow \mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$$

6 parameters

Transition Line

$T_0 = 158$ MeV - crossover temperature at $\mu_B = 0$

$$T'[T_C, \mu_{BC}] = T_0$$

Slope

Choosing μ_{BC} fixes T_C and α_1

$$\alpha_1 = \tan^{-1}\left(\frac{2\kappa_2(T_C)\mu_{BC}}{T_C T'_{,T}}\right)$$

Examples

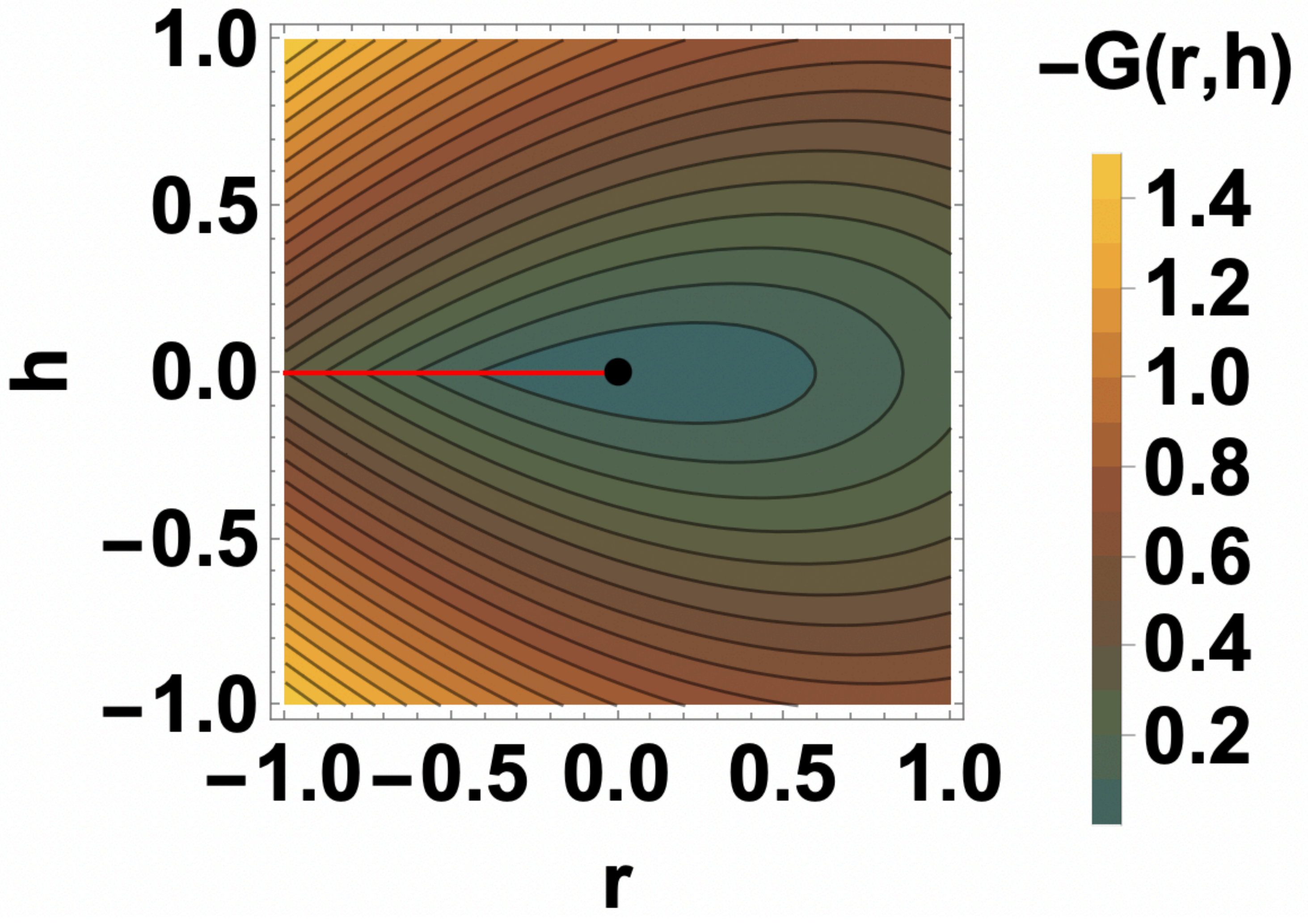
- $\mu_{BC} = 350$ MeV, $T_C = 140$ MeV and $\alpha_1 = 6.6^0$
- $\mu_{BC} = 600$ MeV, $T_C = 94.3$ MeV and $\alpha_1 = 14^0$

BEST COLLABORATION

TExS

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Introducing Critical Point



3D Ising Coordinates

Parameters Choice

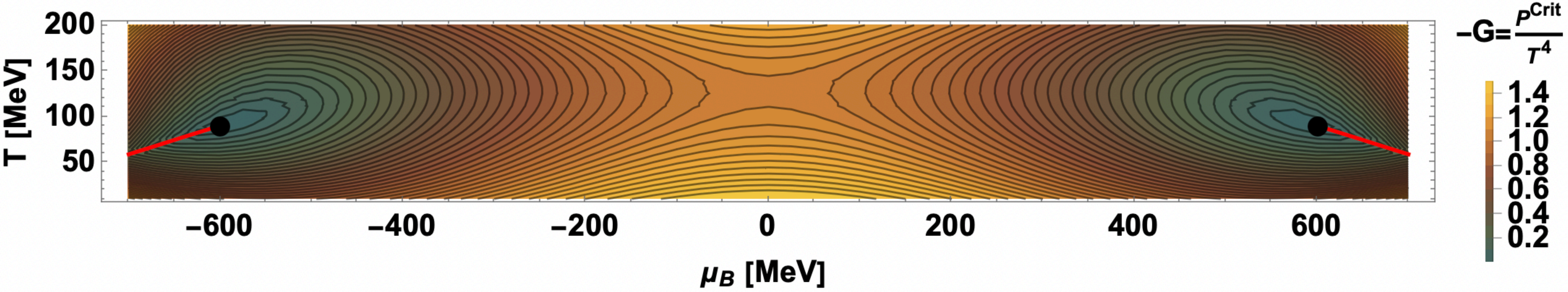
$\mu_{BC} = 350$ MeV, $T_C = 140$ MeV

$T_0 = 158$ MeV, $\alpha_1 = 6.65^0$

$\alpha_{12} = 90^0$, $\alpha_2 = \alpha_1 - \alpha_{12}$

$w = 10$, $\rho = 0.5$

$$T_C \left[1 + \kappa(T_C) \left(\frac{\mu_{BC}}{T_C} \right)^2 \right] = T_0$$



QCD Coordinates

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Part 4: Merging 3D Ising with T′ Expansion (Ising-TExS)

Merging Ising with Lattice (Ising-T ExS)

Full Baryon Density

$$\chi_1^B(T,\mu_B) = \frac{n_B(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

$$T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

Lattice Term

Ising Term

Introducing a Critical Point

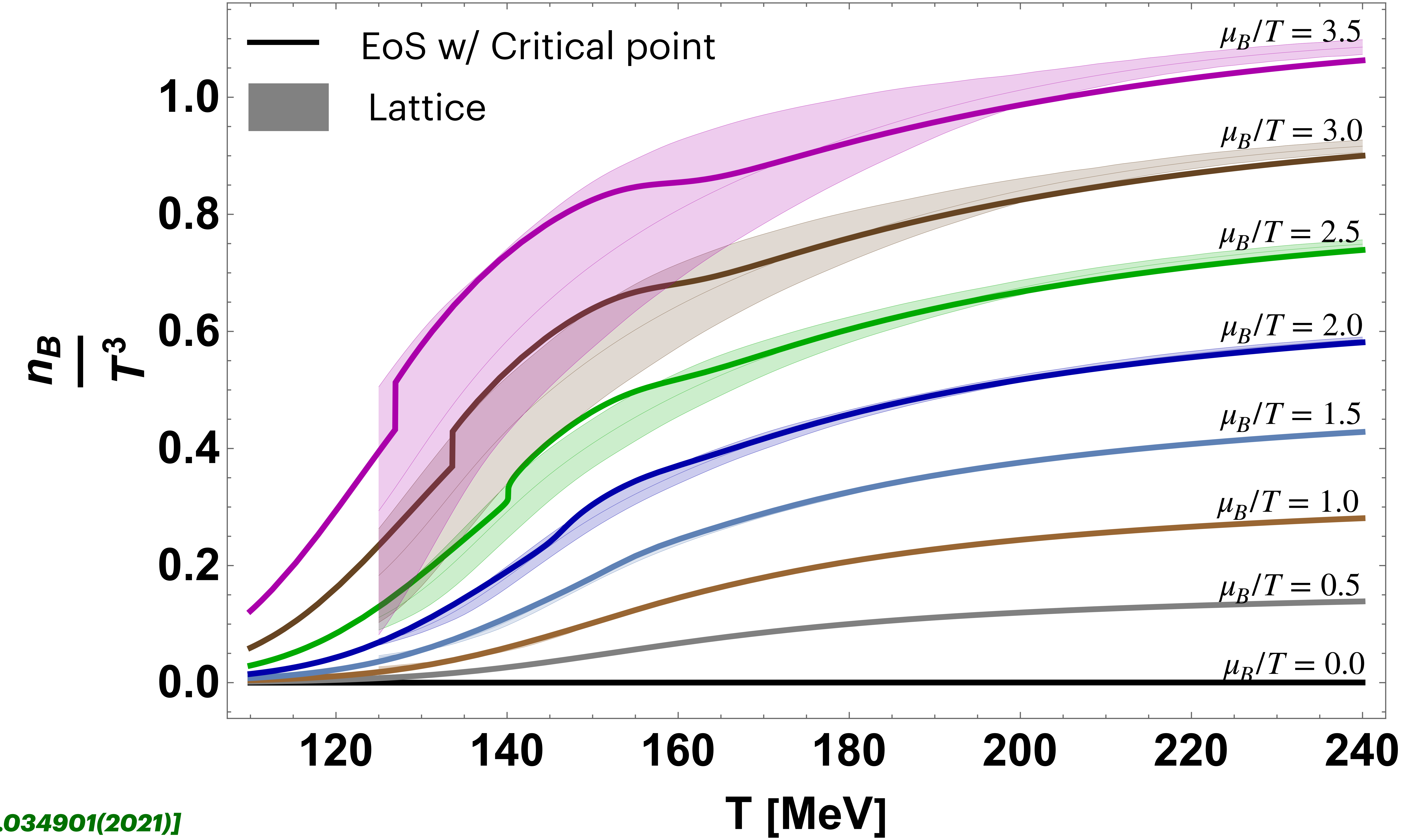
$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)} + \ldots$$

$$Taylor[T'_{crit}, n=2] \approx \left(\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \left[\frac{\partial(n_B^{crit}/T^3)}{\partial(\mu_B/T)}\bigg|_{\mu_B/T=0} + \frac{1}{3!}\frac{\partial^3(n_B^{crit}/T^3)}{\partial(\mu_B/T)^3}\bigg|_{\mu_B/T=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots\right]$$

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Baryon density results

Full Baryon Density at a constant $\frac{\mu_B}{T}$ compared with Lattice



Parameter choice

$\mu_B = 350$ [MeV]

$\alpha_{12} = 90^0$

$w = 2$

$\rho = 2$

[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

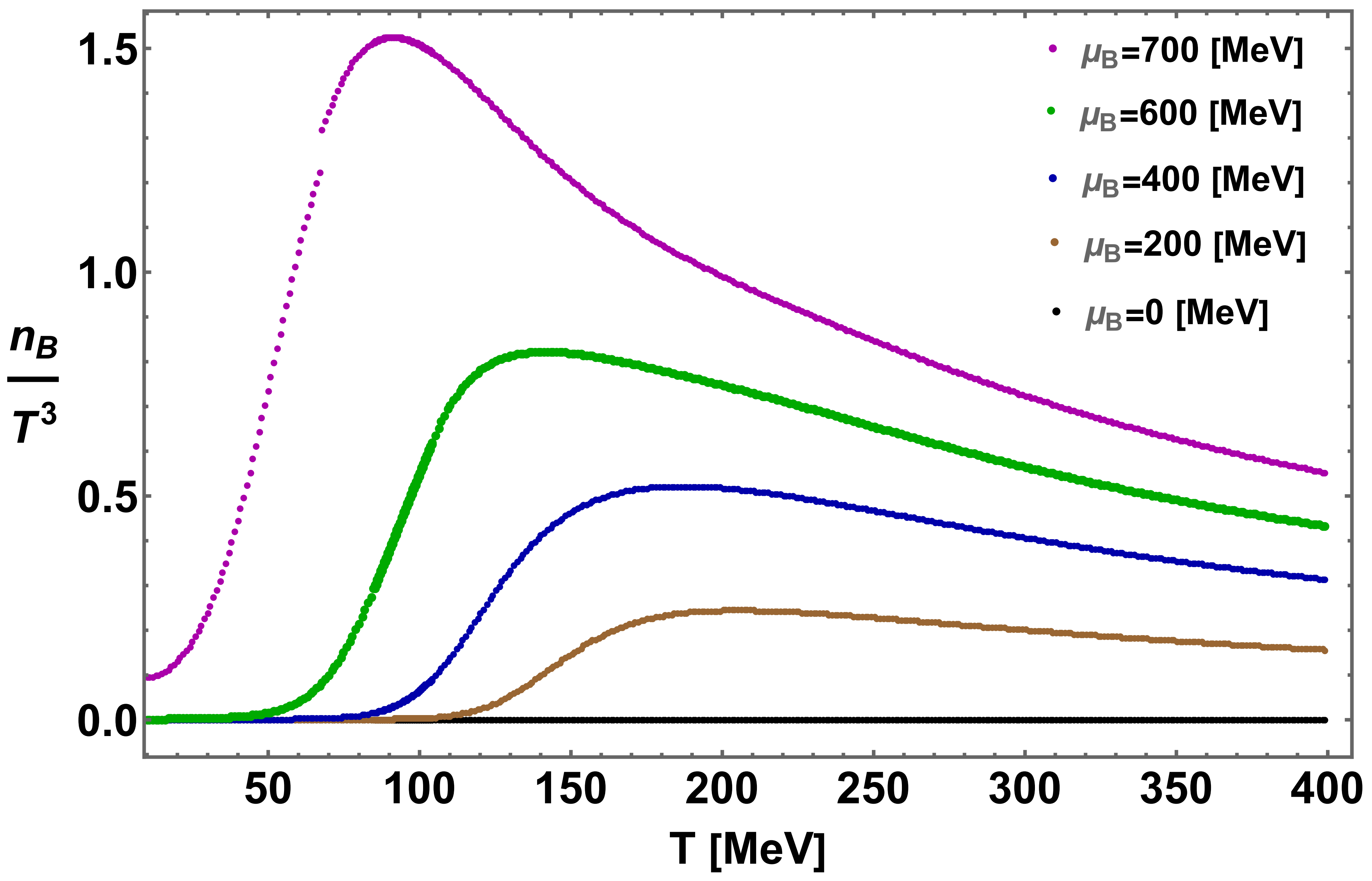
Thermodynamic Observables

Parameter choice

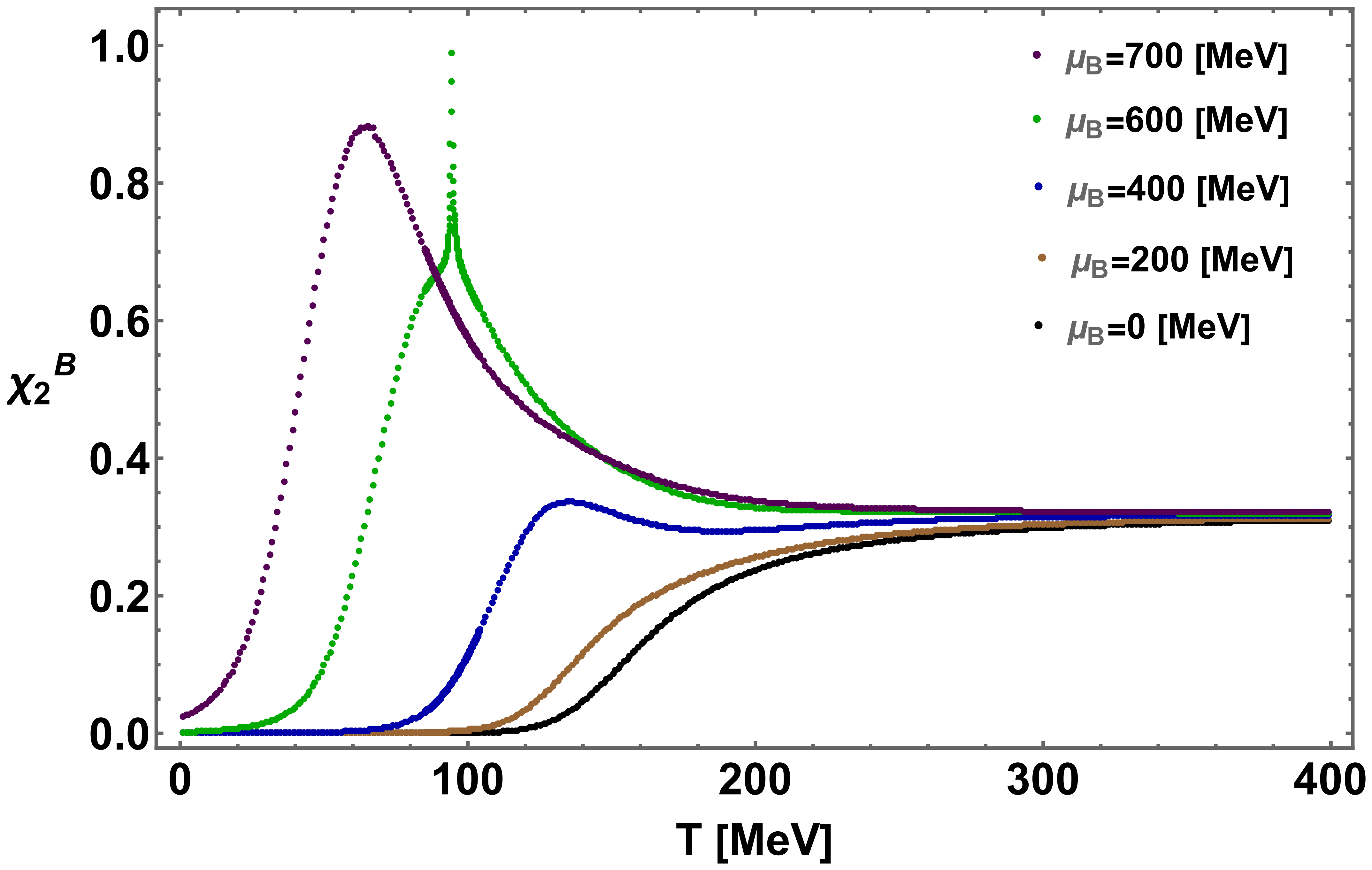
$$\mu_{BC} = 600\text{ MeV}$$
$$T_C = 94.3\text{ MeV}$$
$$\alpha_{12} = \alpha_1 = 14^0$$
$$\alpha_2 = 0^0$$
$$w = 15$$
$$\rho = 0.3$$

$$\chi_2(T,\mu_B) = \left.\frac{\partial(n_B/T^3)}{\partial(\mu_B/T)}\right|_T$$

Baryon Density



Baryon number susceptibility



[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Other Observables

Parameter choice

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

$\alpha_{12} = \alpha_1 = 14^0$

$\alpha_2 = 0^0$

$w = 15$

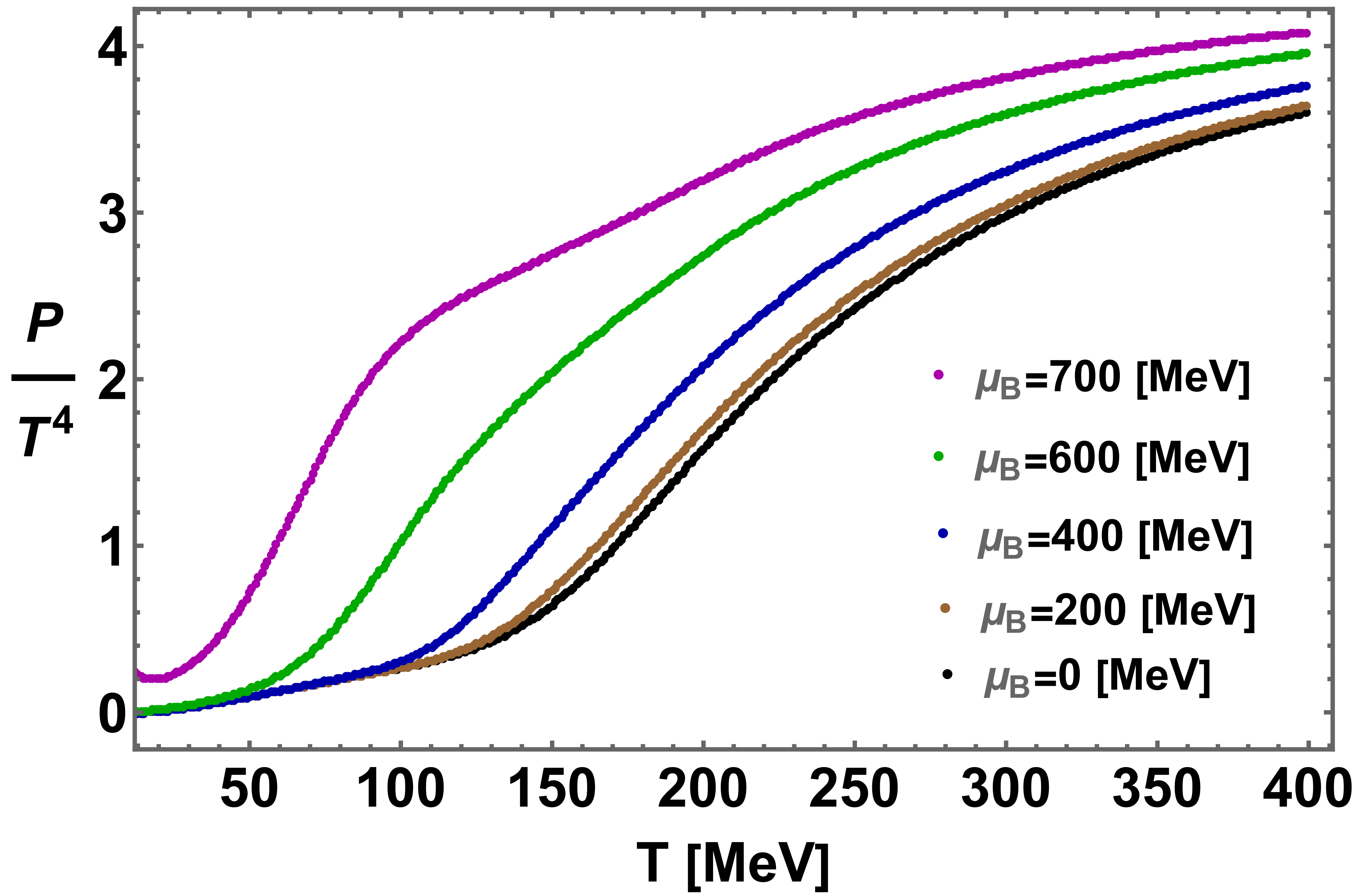
$\rho = 0.3$

$$\frac{P(T,\mu_B)}{T^4} = \chi^B_{0,lat}(T,0) + \frac{1}{T}\int_0^{\mu_B} d\hat{\mu}'_B \frac{n_B(T,\hat{\mu}'_B)}{T^3}$$

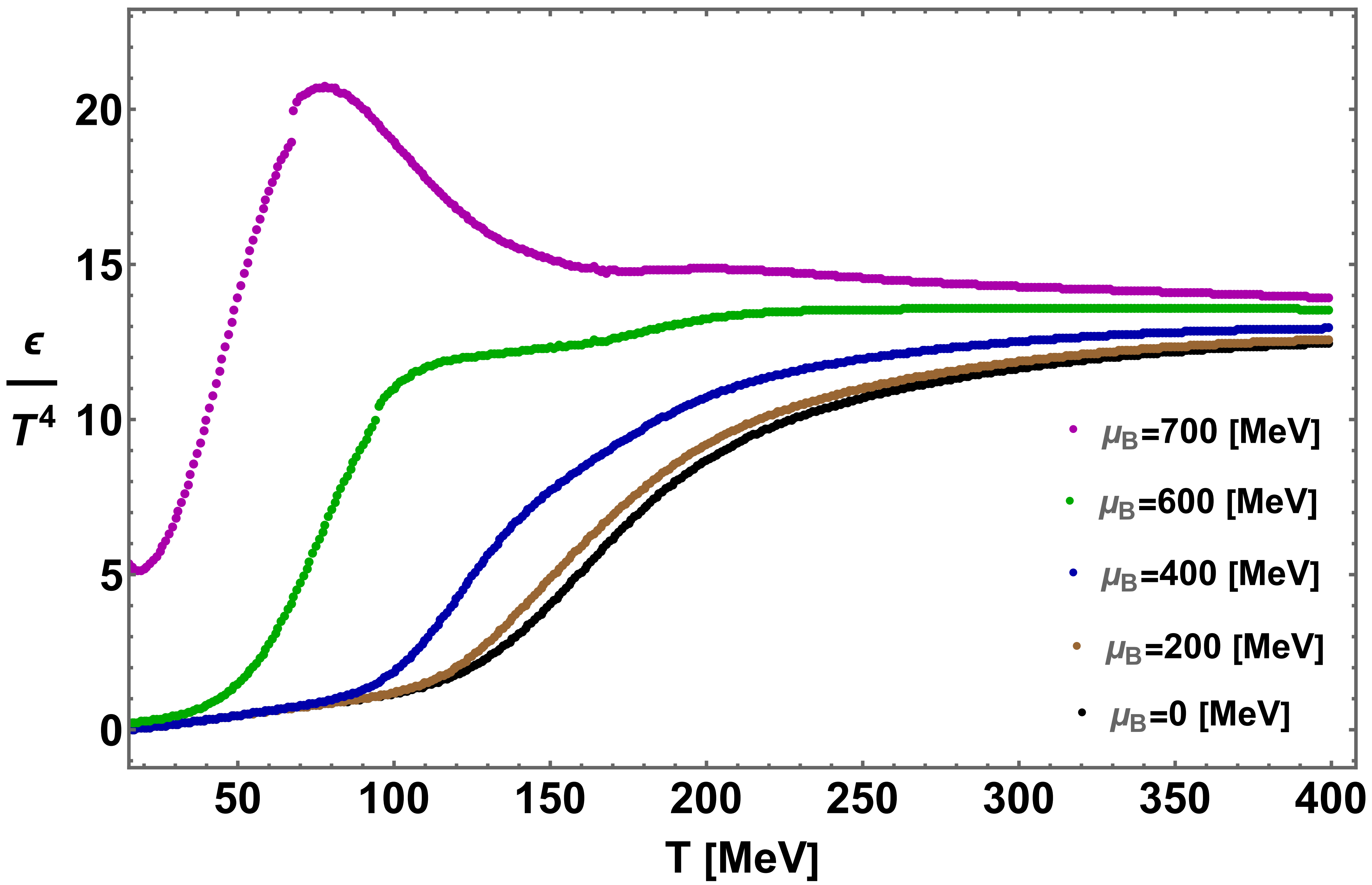
$$\frac{s(T,\mu_B)}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial T}\right)\bigg|_{\mu_B}$$

$$\frac{\epsilon(T,\mu_B)}{T^3} = \frac{s}{T^3} - \frac{P}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3}$$

Pressure



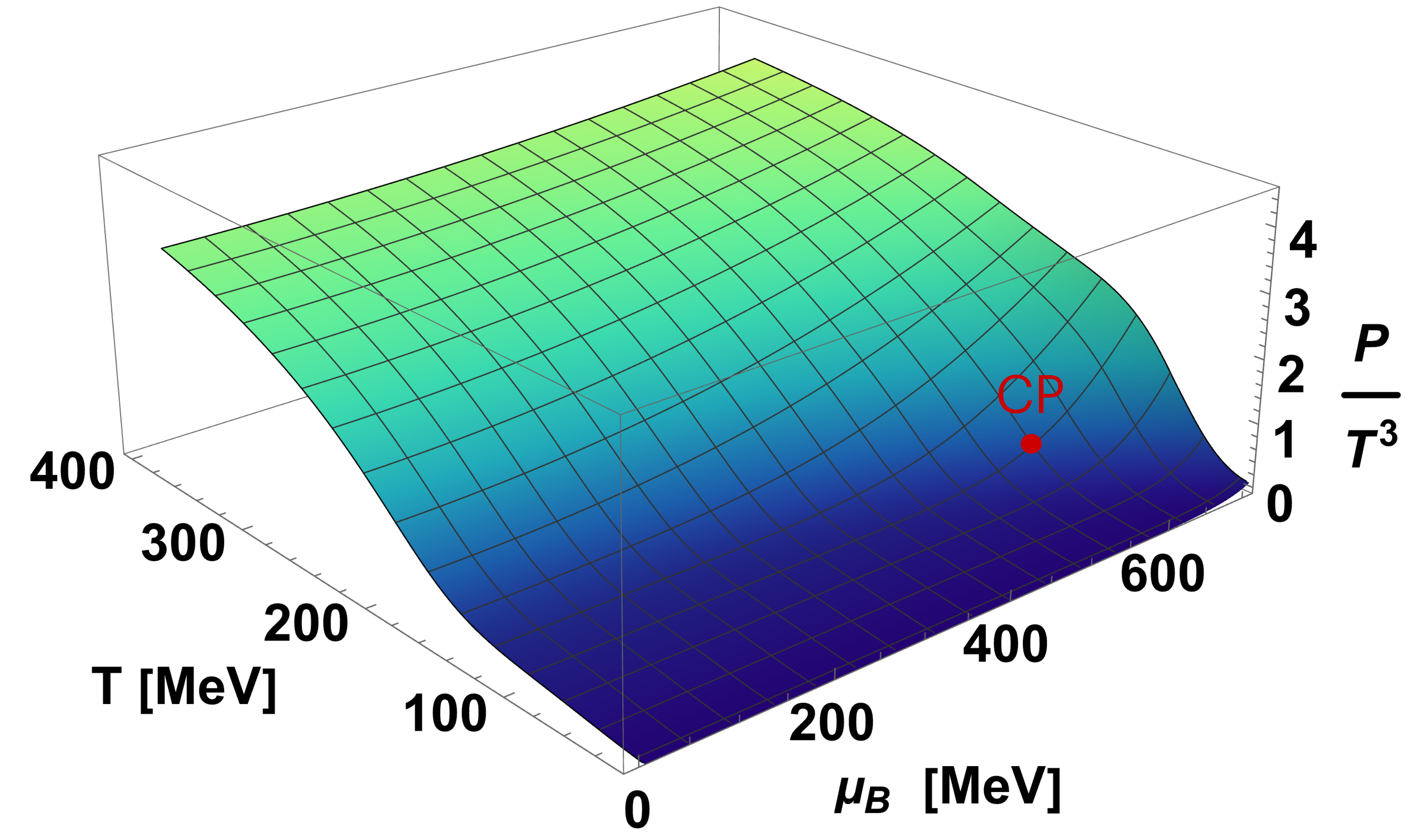
Energy density



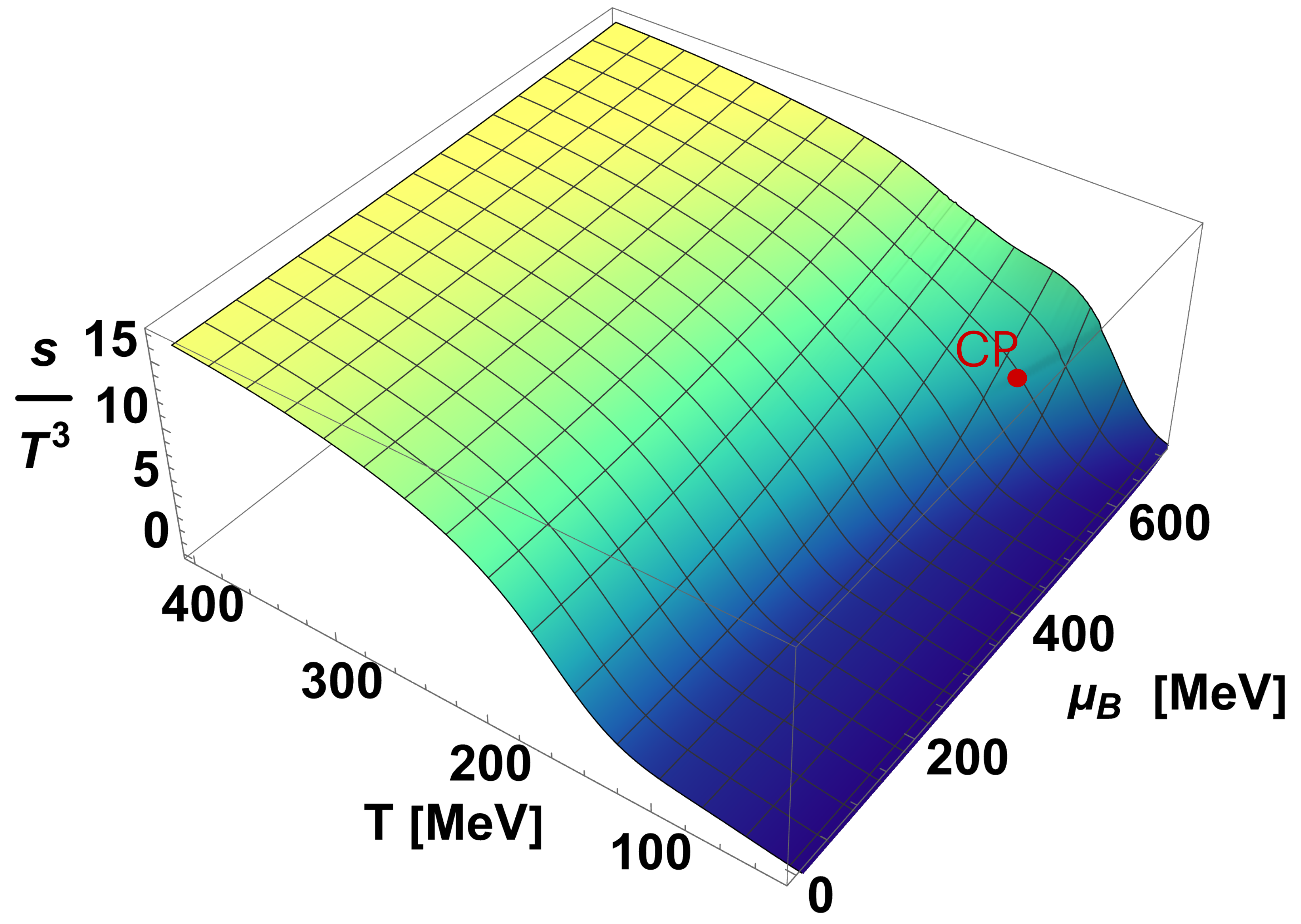
[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Thermodynamic Observables

Pressure



Entropy density



Parameter choice

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

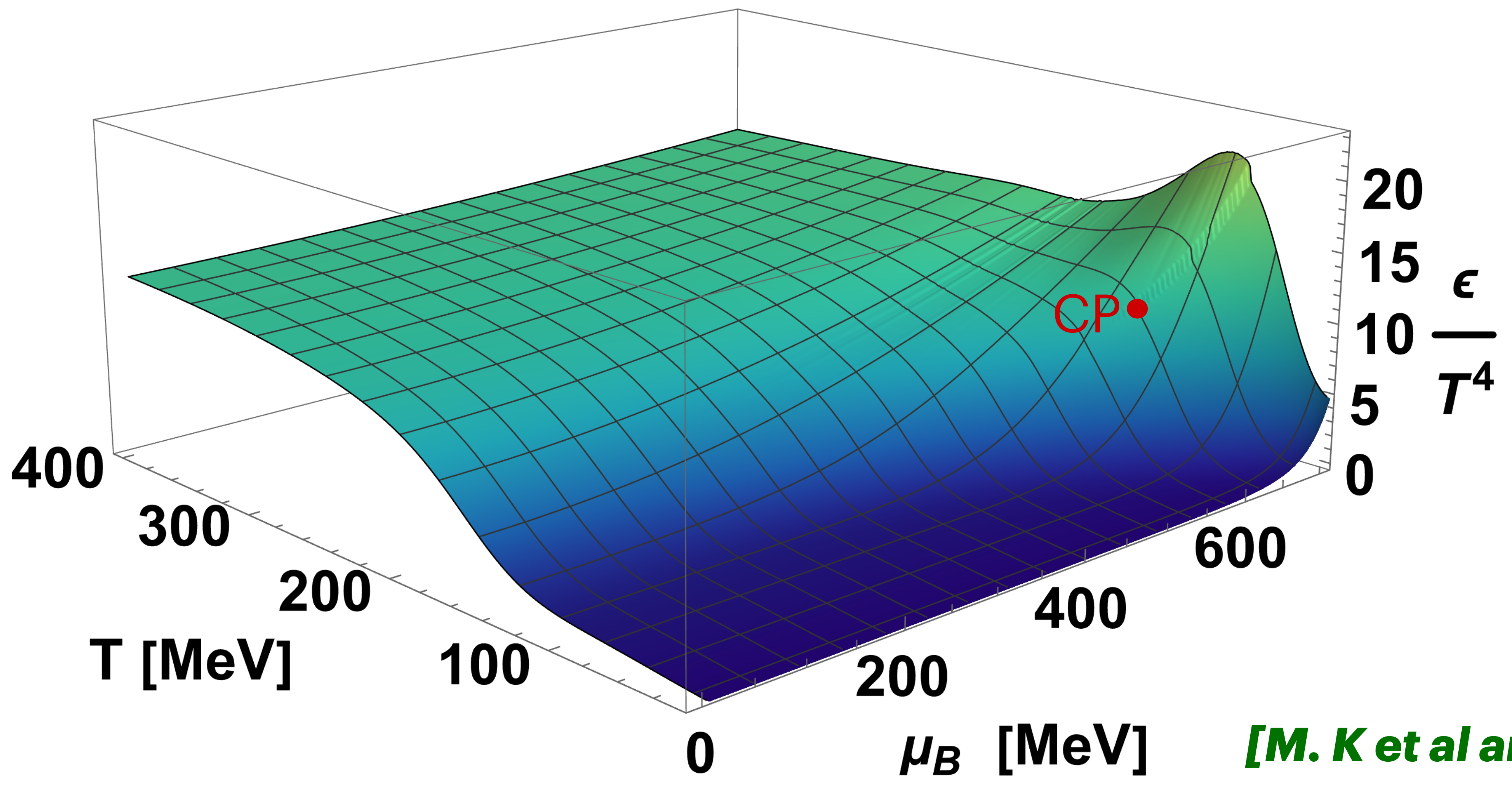
$\alpha_{12} = \alpha_1 = 14^0$

$\alpha_2 = 0^0$

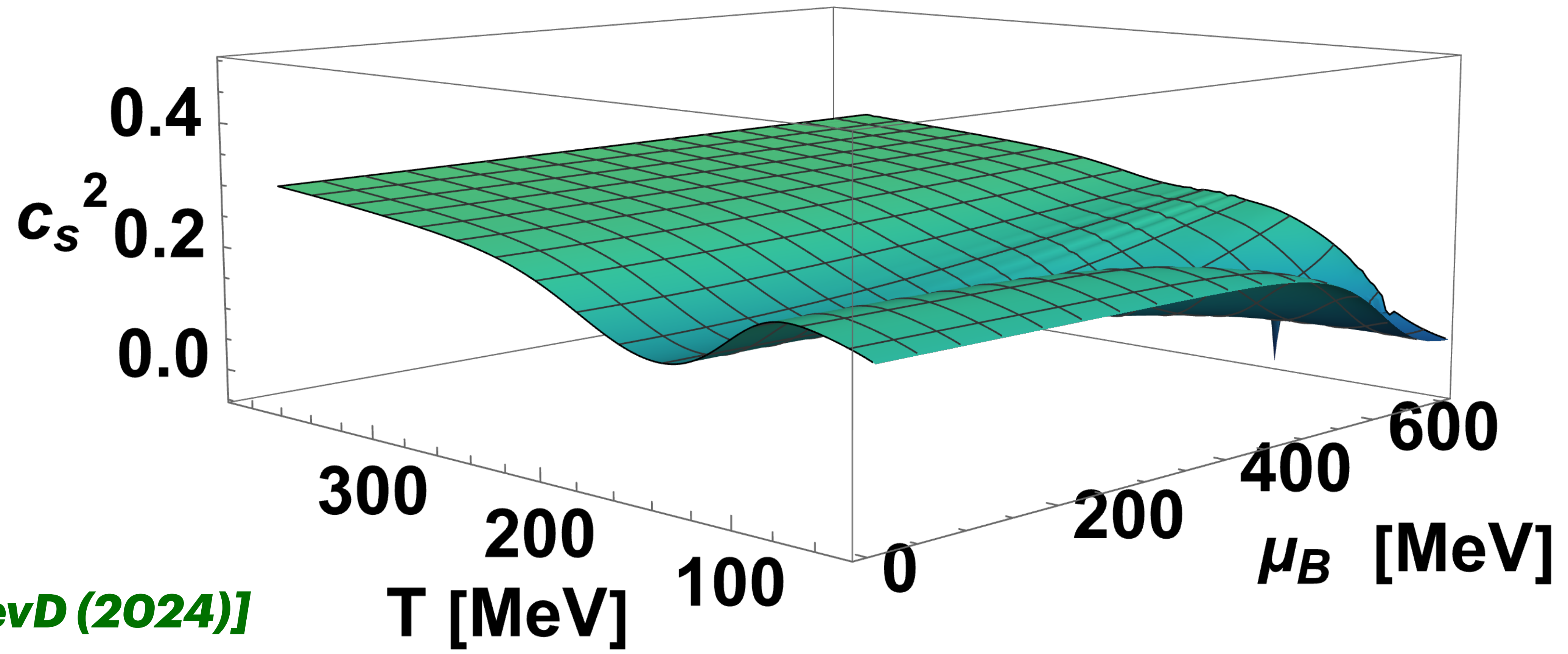
$w = 15$

$\rho = 0.3$

Energy Density



Speed of Sound



[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Part 5: Constraints on the EoS

Known constraints on the EoS

- Lattice QCD disfavor $\mu_{BC} < 300$ MeV

[Borsányi, S et al PhysRevL. 125, 052001(2020)]

- Choosing μ_{BC} fixes T_C and α_1
- α_{12} is fixed by physical quark mass requirement $(\alpha_{12} = \alpha_1)$

[Pradeep, M. S., & Stephanov, M PhysRevD 100(5), 056003.(2019)]

Stability and causality

- w and ρ are fixed stability and causality

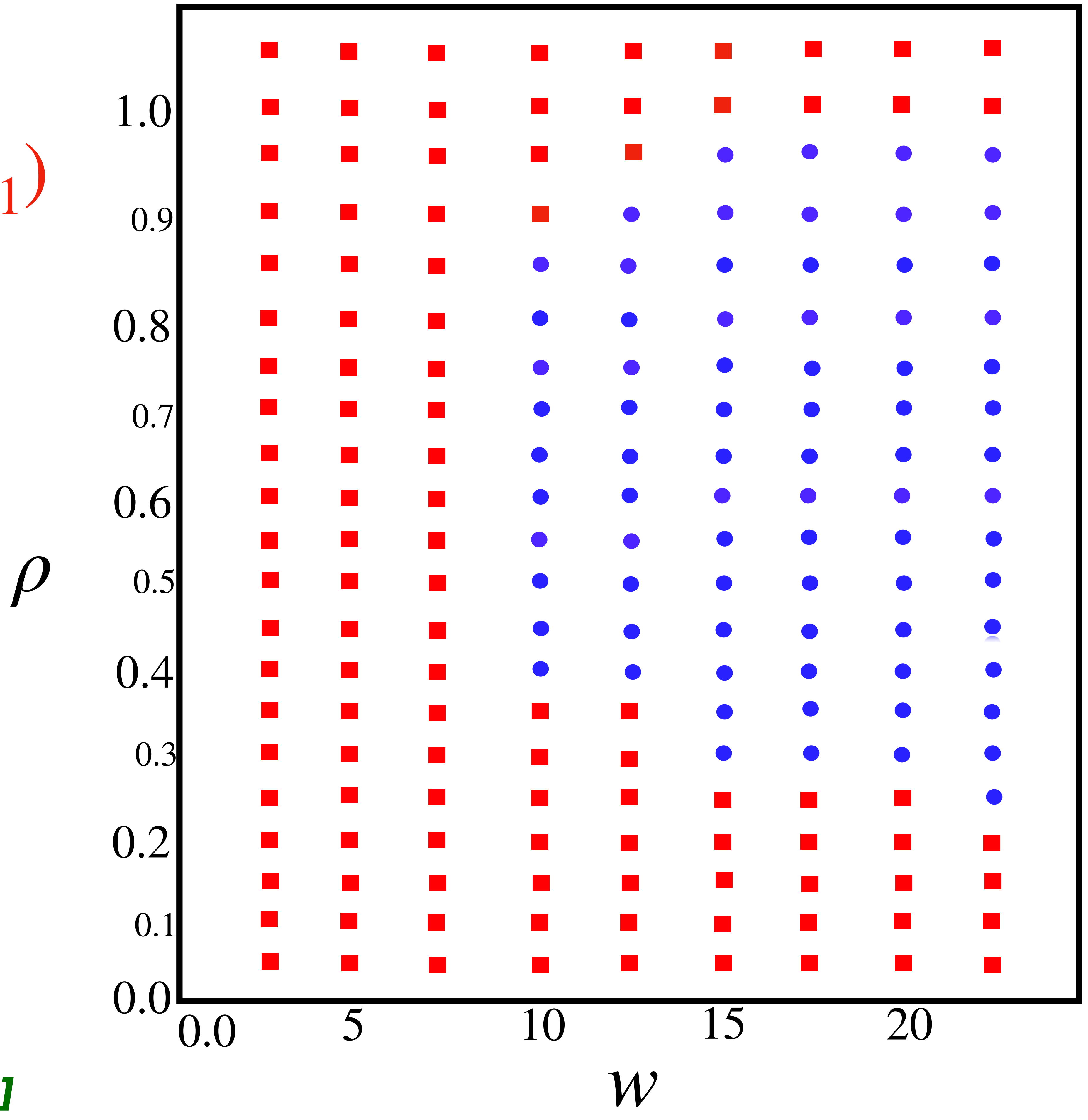
$$c_v = \left(\frac{\partial s}{\partial T}\right)\bigg|_{n_B} > 0$$

$$\chi_2(T, \mu_B) = \left(\frac{\partial n_B}{\partial \mu_B}\right)\bigg|_{T} > 0$$

$$0 < c_s^2(T, \mu_B) < 1$$

$\mu_{BC} = 600$ MeV

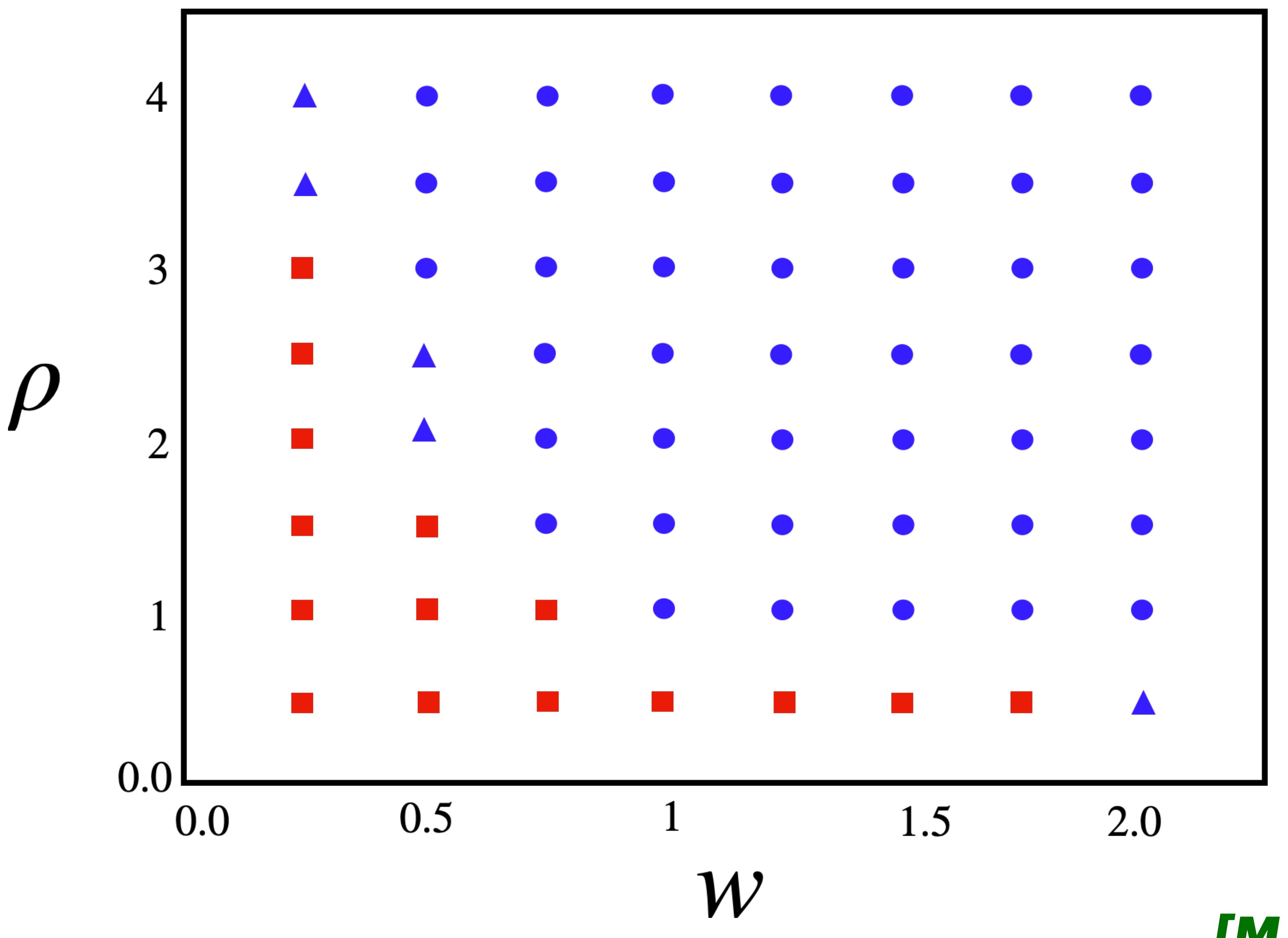
$\alpha_{12} = \alpha_1$



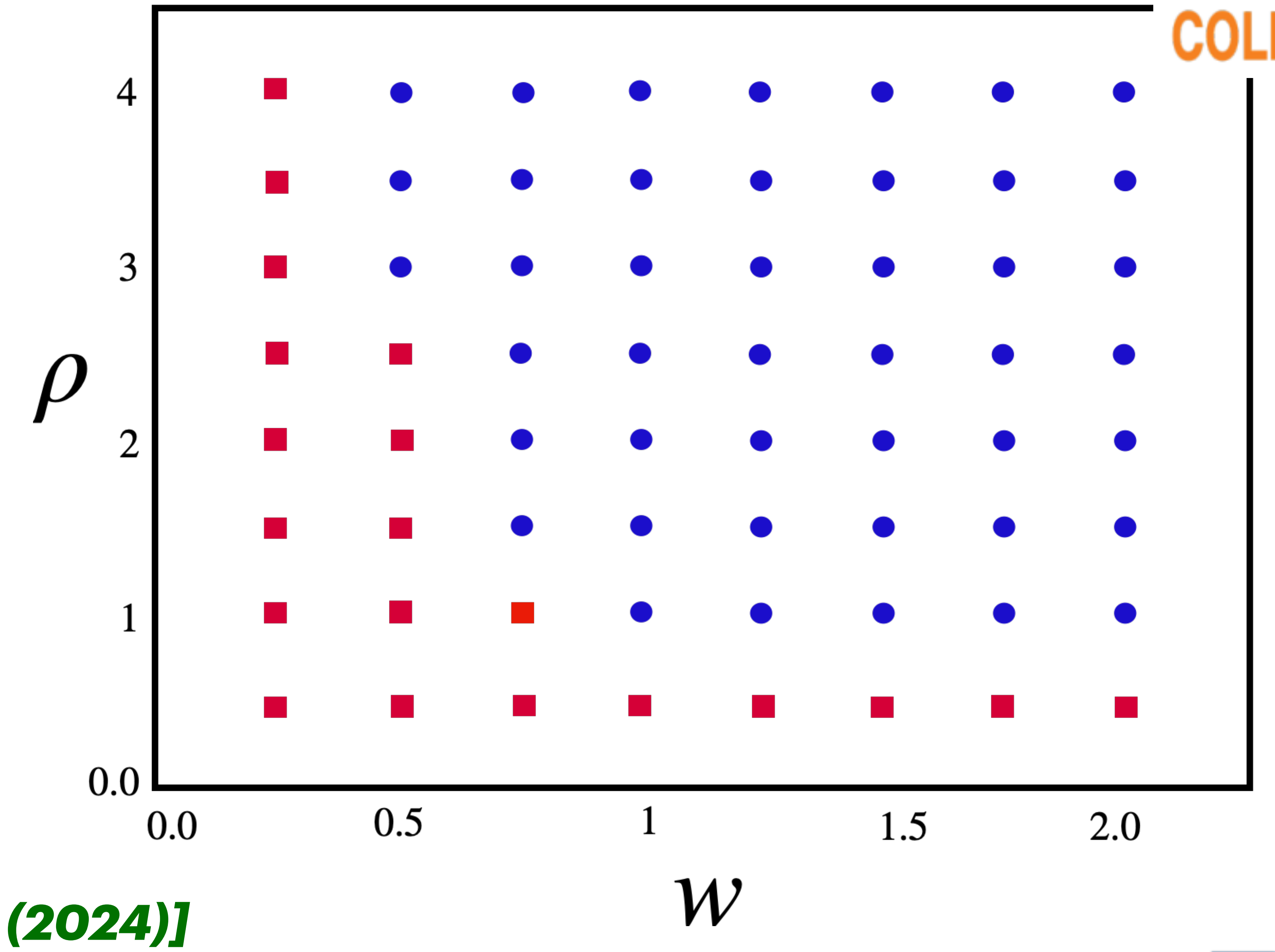
[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

Comparison of Ising-TEXS with BEST EoS

Ising-T ExS



Taylor Expansion



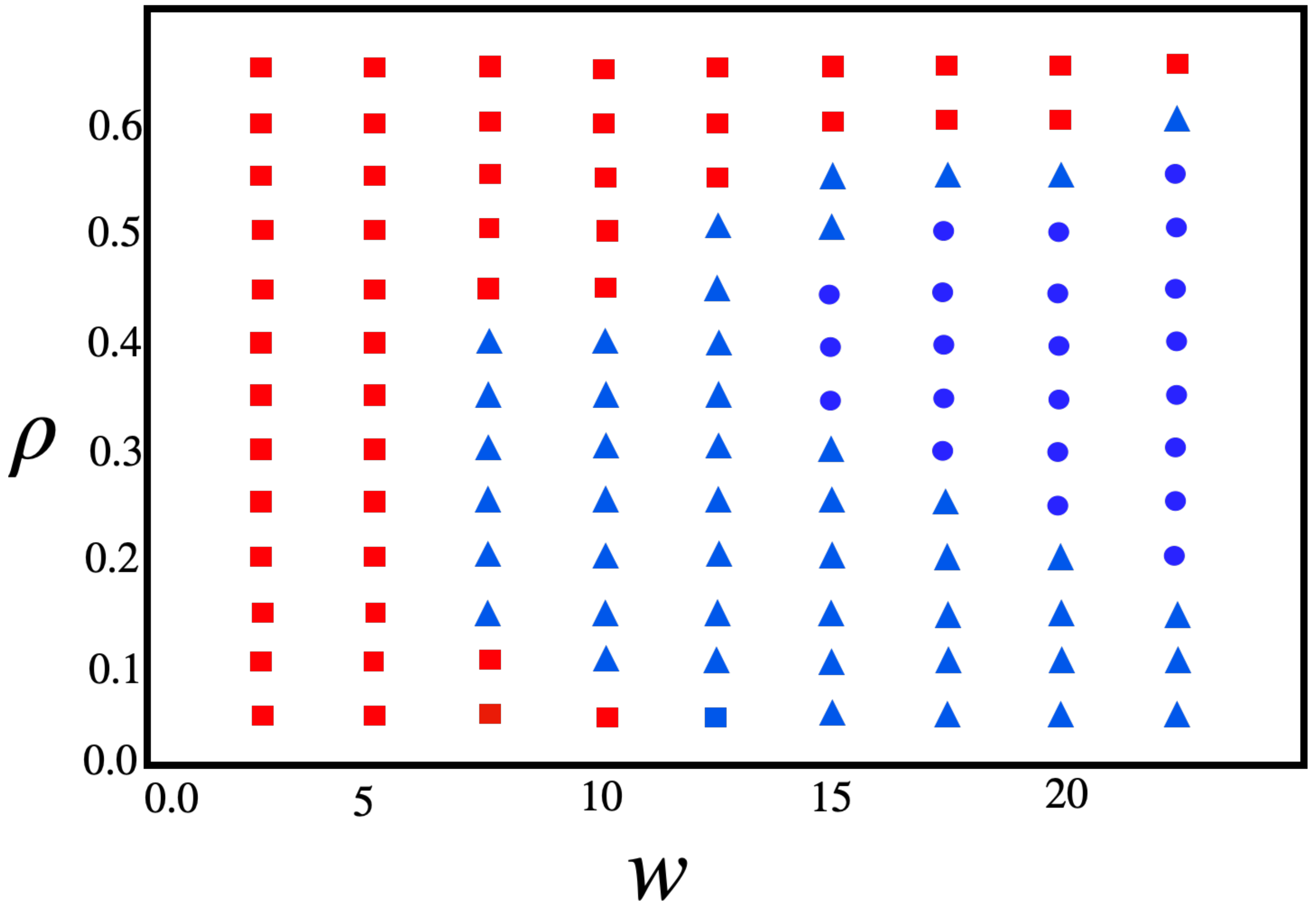
BEST COLLABORATION

$\mu_{BC} = 350$ MeV

$\alpha_{12} = 90$

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

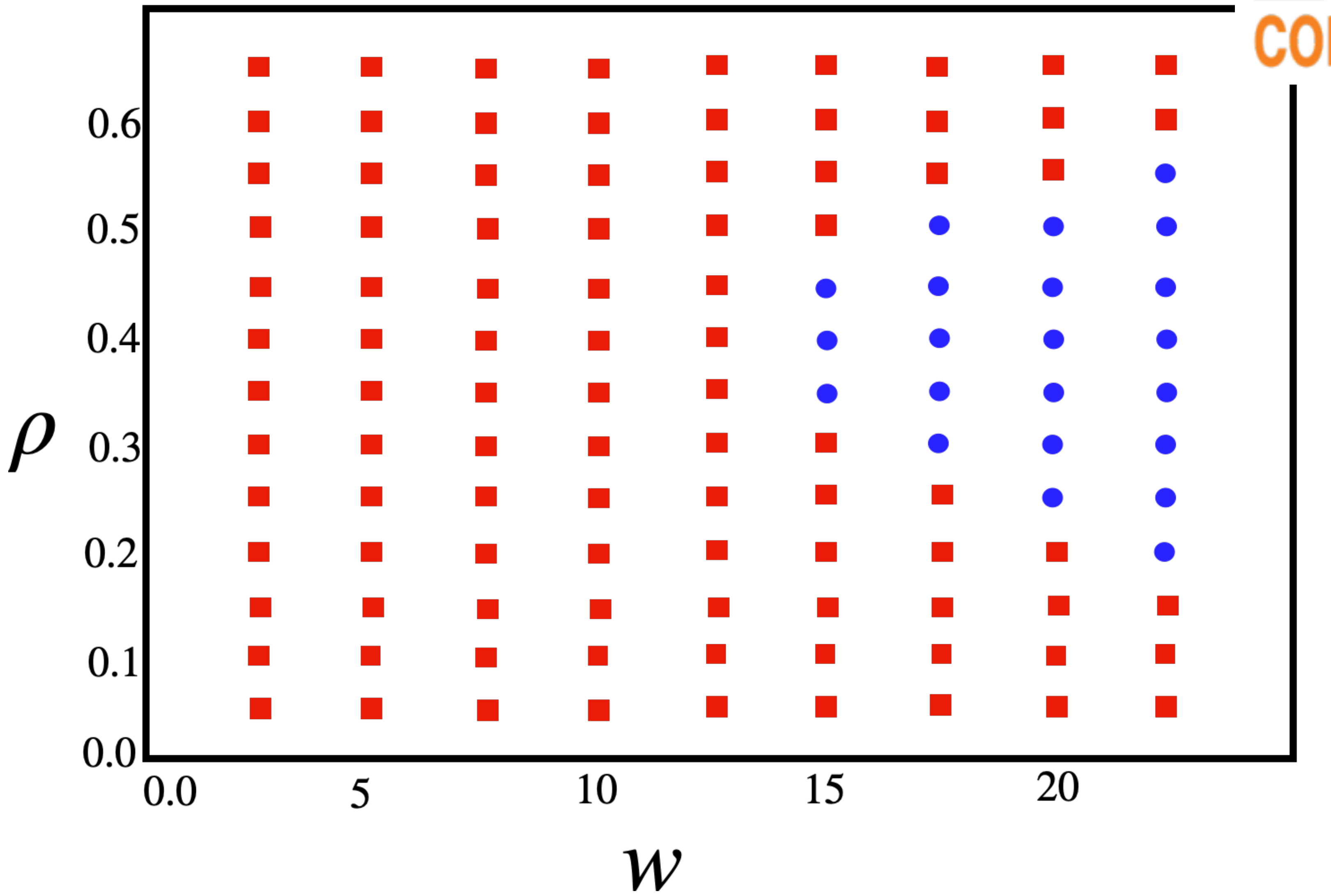
Ising-T ExS



$\mu_{BC} = 350$ MeV

$\alpha_{12} = \alpha_1$

Taylor Expansion



BEST COLLABORATION

Summary and Conclusion

- We provide an **enhanced coverage** for family of EoS with a 3D Ising critical point up to $\mu_B = 700\ MeV$ and match lattice at low μ_B.

- Ising TExS EoS incorporates **charge conjugation** symmetry inbuilt directly from the Ising -QCD mapping.

- Ising TExS mapping can be constrained to reproduce expectations based on **physical quark masses.**

- Ising TExS has adjustable parameters and can be used as input in hydrodynamical simulations to compare with the data from the Experiment. (Beam Energy Scan II)

Disclaimer! : We do not predict the location of the critical point

Thank you for listening !

Back up!

Important relations

Relationship of TExS with BEST Mapping

BEST COLLABORATION

$$\mu_{BC}, T_C, \alpha'_{12}, w', \rho' \longrightarrow \mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$$

6 parameters

$$\tan\alpha'_{12} = \tan\alpha_1 - \tan\alpha_2, \qquad \rho' = \rho\frac{\cos^2\alpha_1}{\sqrt{(\cos\alpha_1\cos\alpha_2)^2 + (\sin\alpha_{12})^2}} \qquad w' = w\frac{1}{\cos\alpha_1}\sqrt{(\cos\alpha_1\cos\alpha_2)^2 + (\sin\alpha_{12})^2}$$

[M. K et al arXiv:2402.08636v1]

[Parotto et al PhysRevC.101.034901(2020)]

Strength of the discontinuity

leading singular behavior of specific heat at constant pressure cp

$$cp = T^3\left(\frac{(s_c/n_c)\sin\alpha_1 - \cos\alpha_1}{w\sin\alpha_{12}}\right)^2 G_{hh}\left(1 + \mathcal{O}(r^{\beta\delta-1})\right)$$

$w\sin\alpha_{12}$ -Controls the strength of the jump

G_{hh} – order parameter in Ising Model

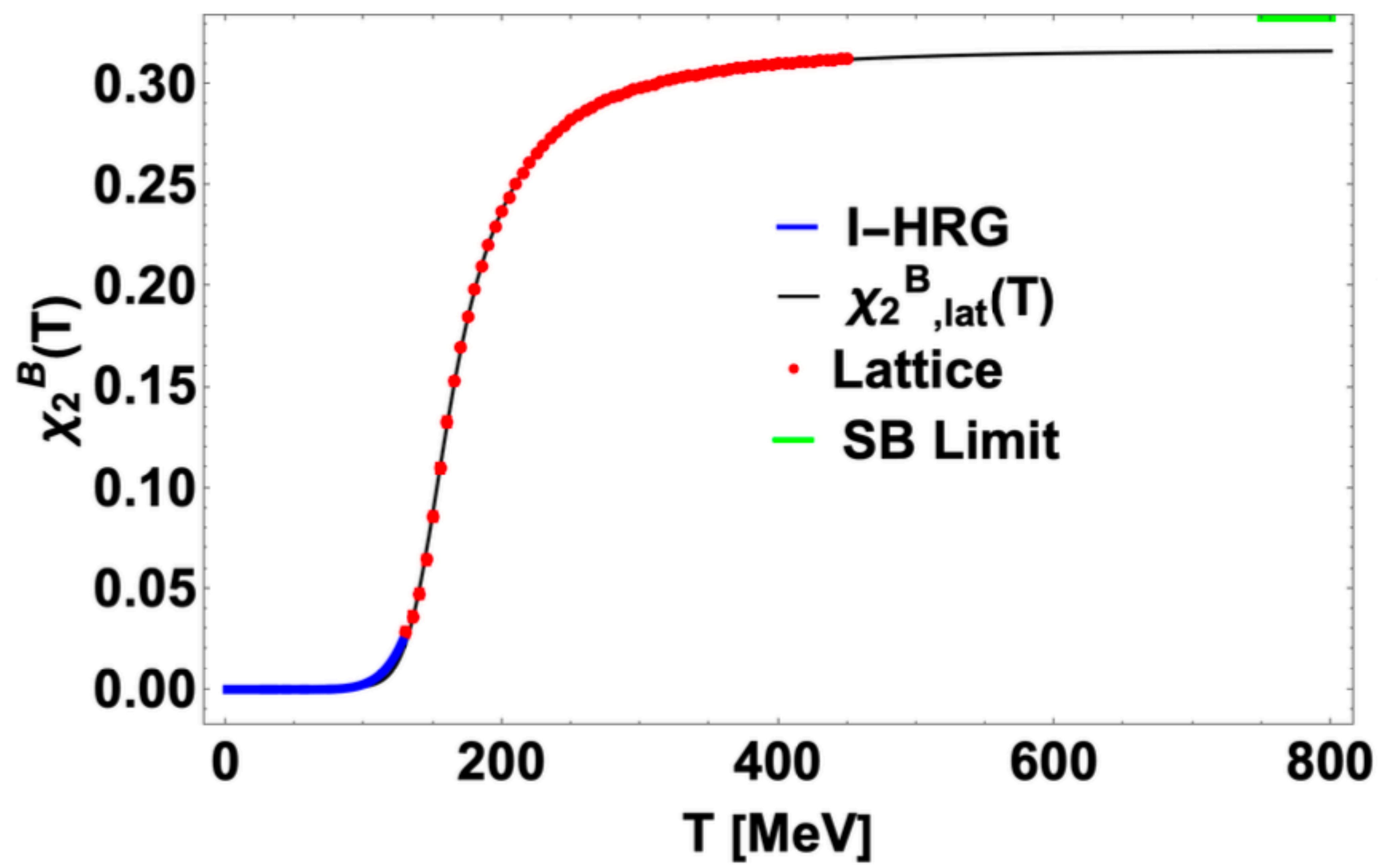
Lattice data: Parametrization

- To have a smooth temperature description from 25 MeV $< T < 800$ MeV,
  We parameterize lattice data and merge with HRG

$$\chi^B_{2,\mathrm{lat}}(T) = \left(\frac{2m_p}{\pi x}\right)^{3/2} \frac{e^{-m_p/x}}{1+\left(\frac{x}{d_1}\right)^{d_2}} + d_3 \frac{e^{-d_4^2/x^2 - d_5^4/x^4}}{1+\left(\frac{x}{d_1}\right)^{-d_2}}$$

$x = \dfrac{T}{200 \text{ MeV}}$ $\quad d_i$ - fitting parameters

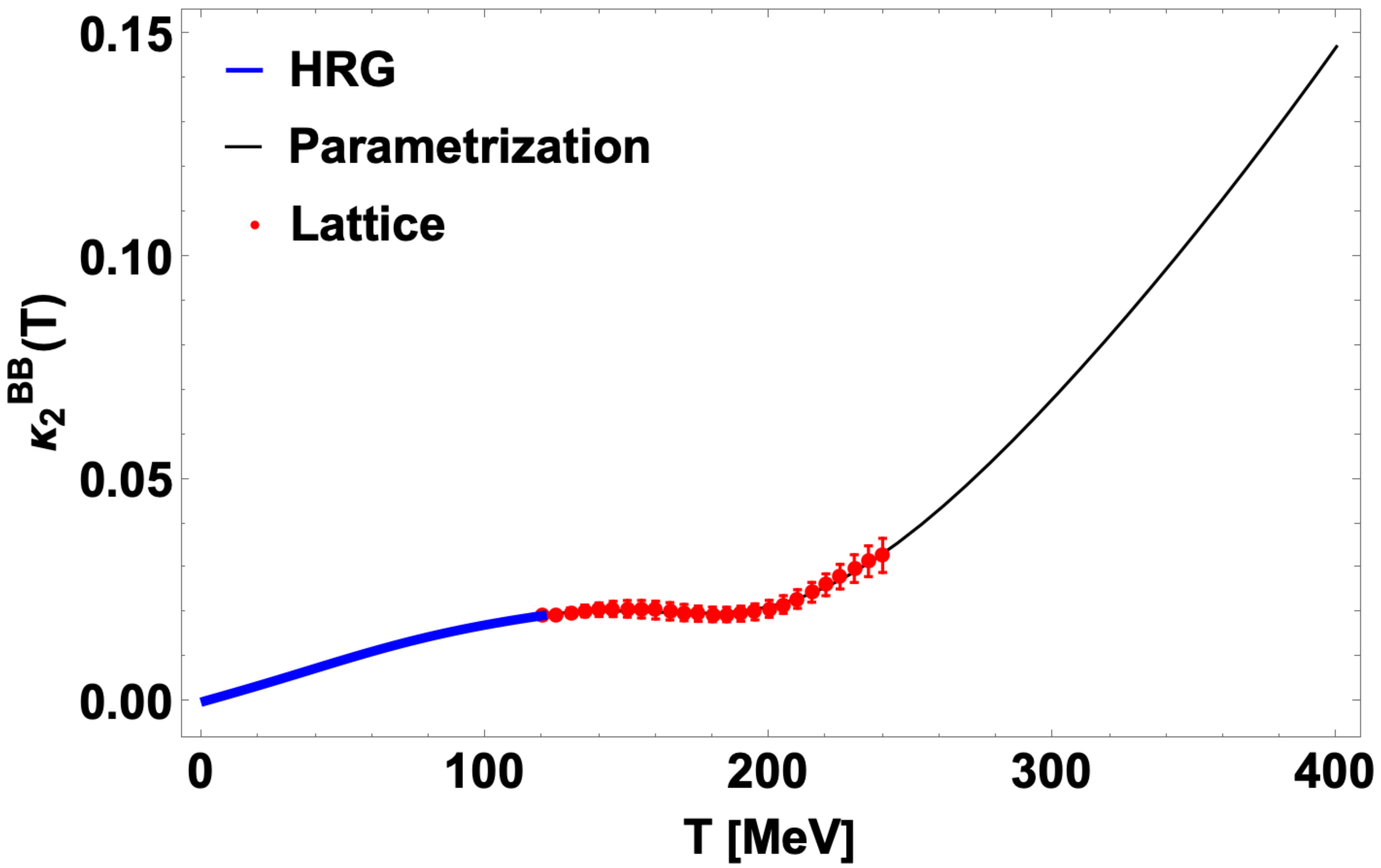
m_p - proton mass (in units of 200 MeV)



[M. K et al arXiv:2402.08636v1]

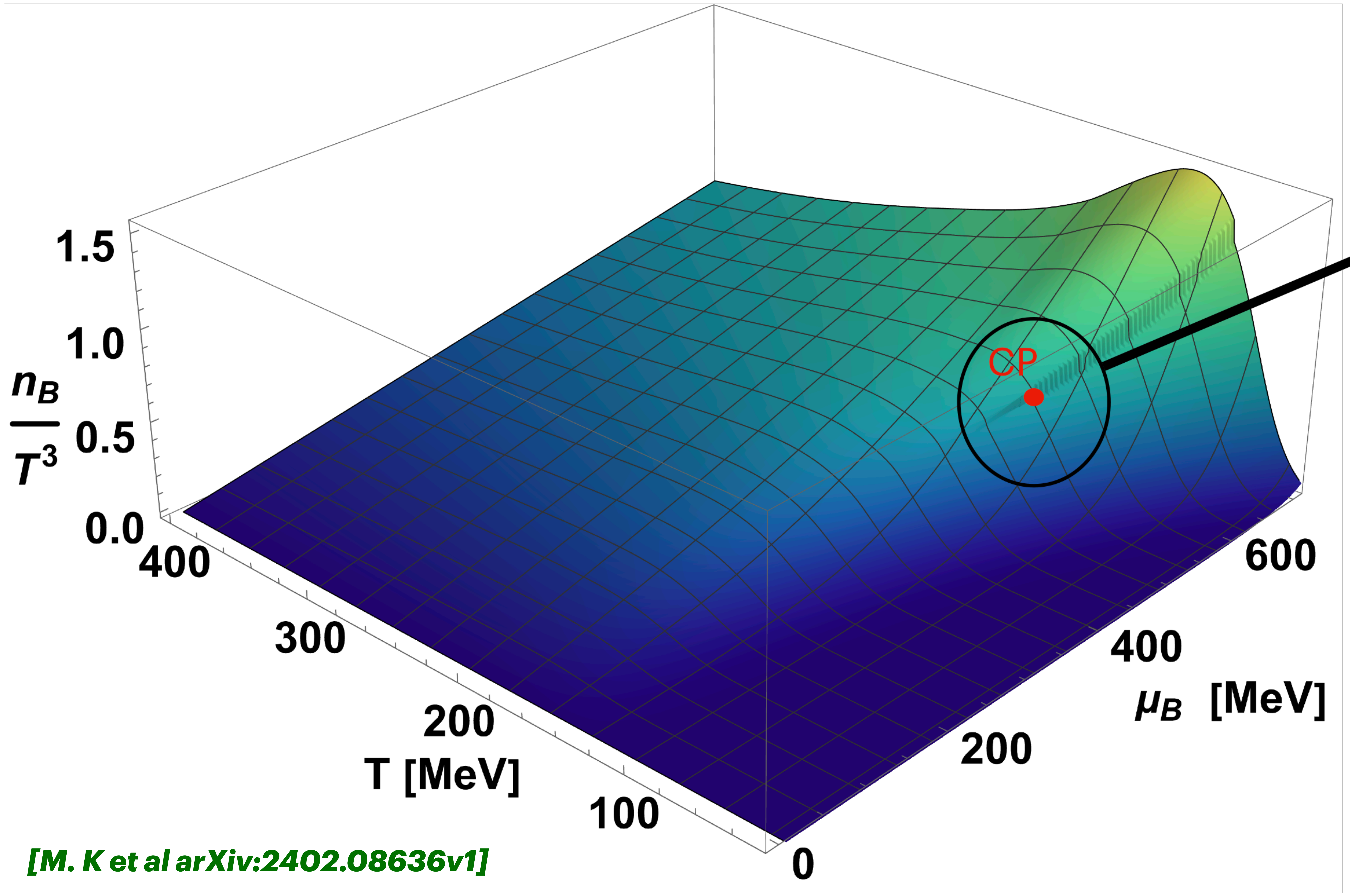
$$\kappa_2^{BB}(T) = \frac{a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5}{b_0 + b_1 x + b_2 x^2 + a_5/A\, x^3}$$

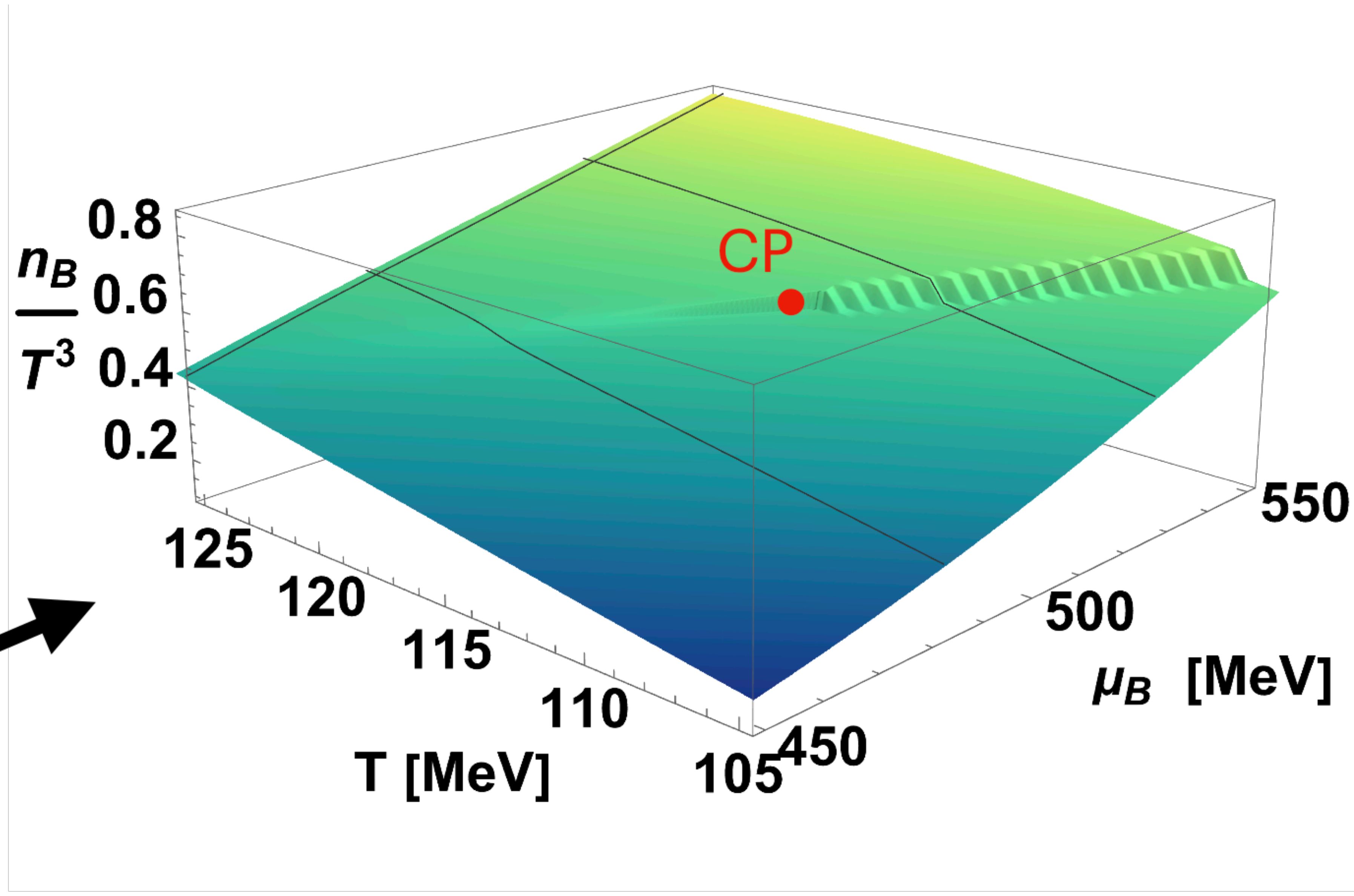
$x = \dfrac{T}{200 \text{ MeV}}$ $\quad d_i$ - fitting parameters



Thermodynamic Observables

Baryon Density n_B/T^3





Parameter choice

$$\mu_{BC} = 500 \text{ MeV}$$

$$T_C = 117 \text{ MeV}$$

$$\alpha_{12} = \alpha_1 = 11^0$$

$$\alpha_2 = 0^0$$

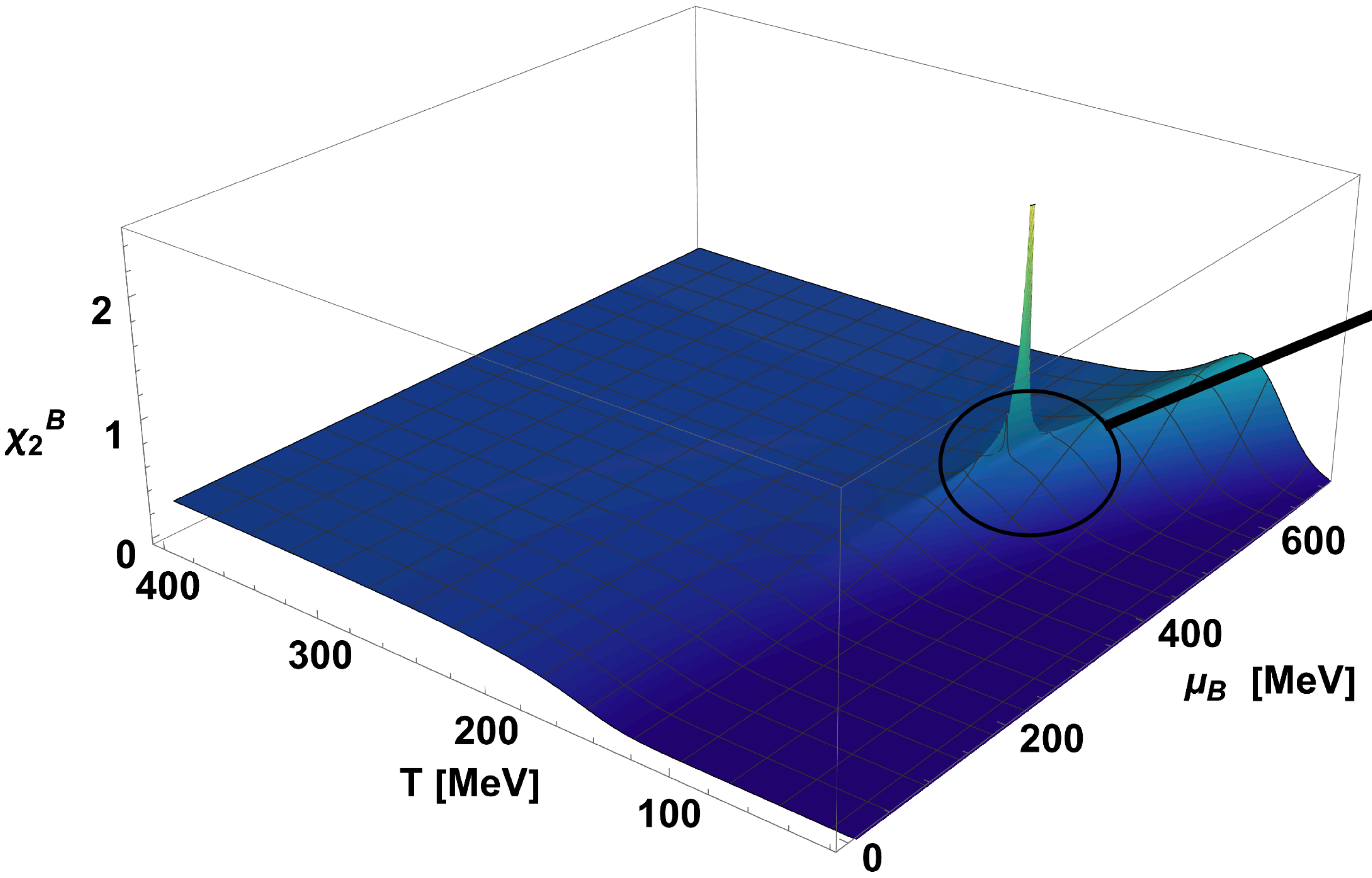
$$w = 15$$

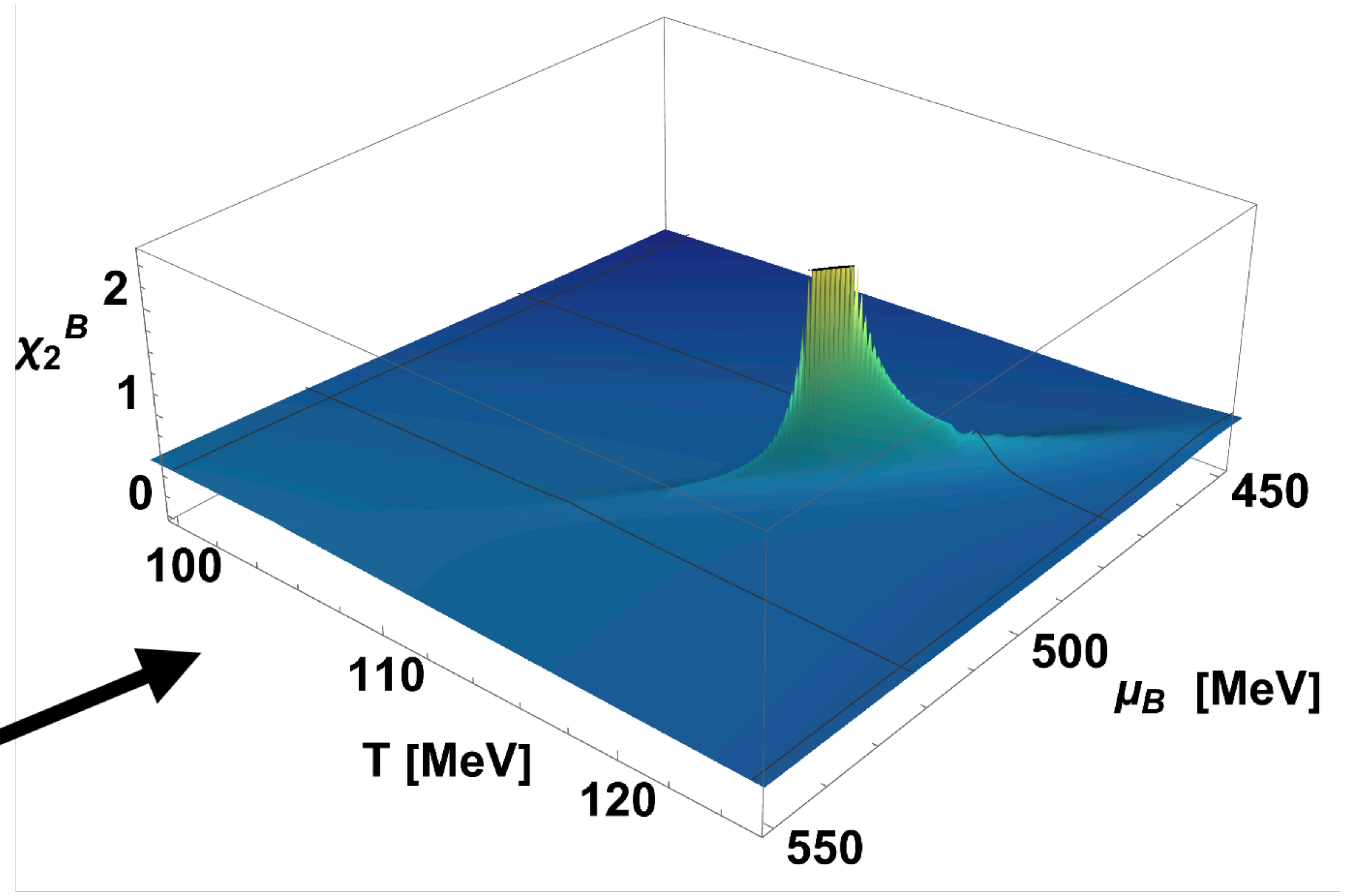
$$\rho = 0.3$$

[M. K et al arXiv:2402.08636v1]

Thermodynamic Observables

Baryon number susceptibility χ_2^B





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