4D-TExS: A Novel 4D Lattice-based QCD Equation of State with Extended Density Range

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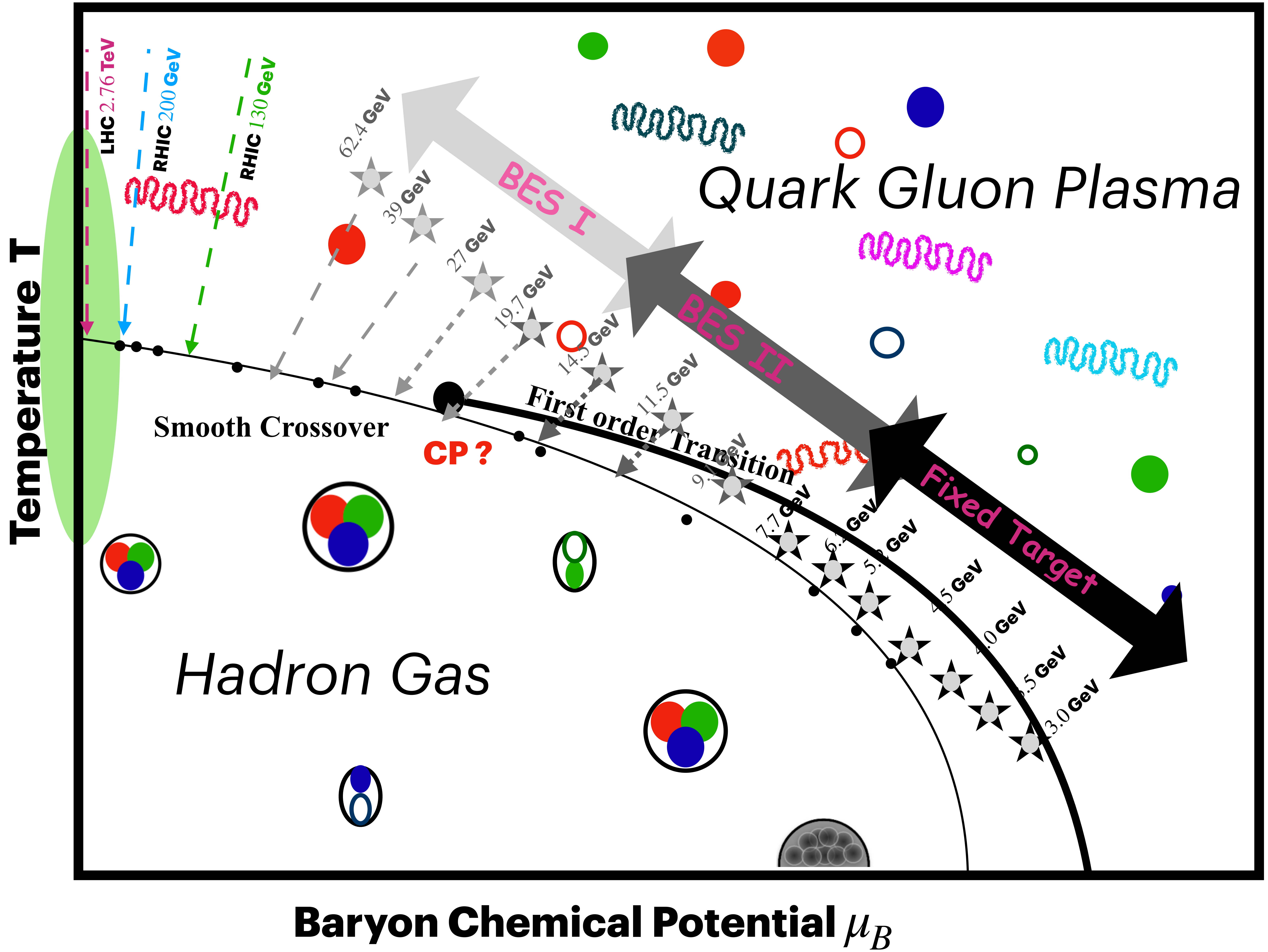
QCD Phase Diagram



Experiments

- Finite density physics is achieved by lowering the $\sqrt{S_{NN}}$ in **BES program**
- Other experiments FAIR, NICA,J-PARC

Theoretical interpretation

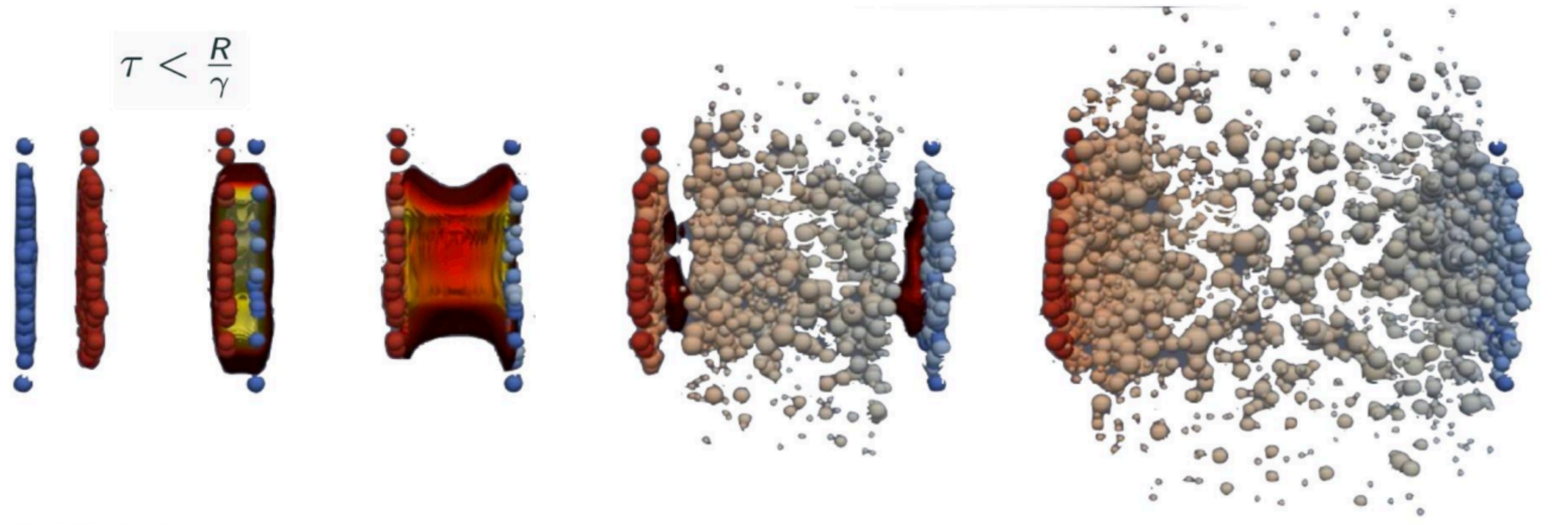
- **Transport models (Hydro)** model fireball evolution in heavy-ion collisions and matter dynamics in neutron star mergers.
- The **Equation of State (EoS)** is essential in hydro, to describe thermodynamics and evolution.

The EoS must capture relevant physics and cover much of the phase diagram

Why we need 4D EoS (T, μ_B, μ_Q, μ_S)

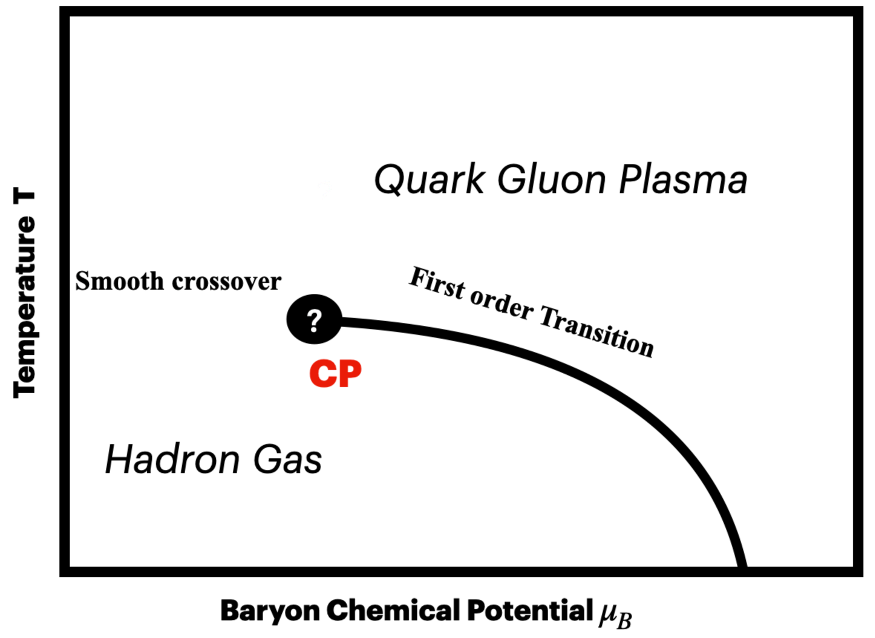
Heavy Ion collision, HI

- Net electric charge Q is non-zero
- To describe **kaons, hyperons** and **strange quark matter**, finite μ_S is necessary



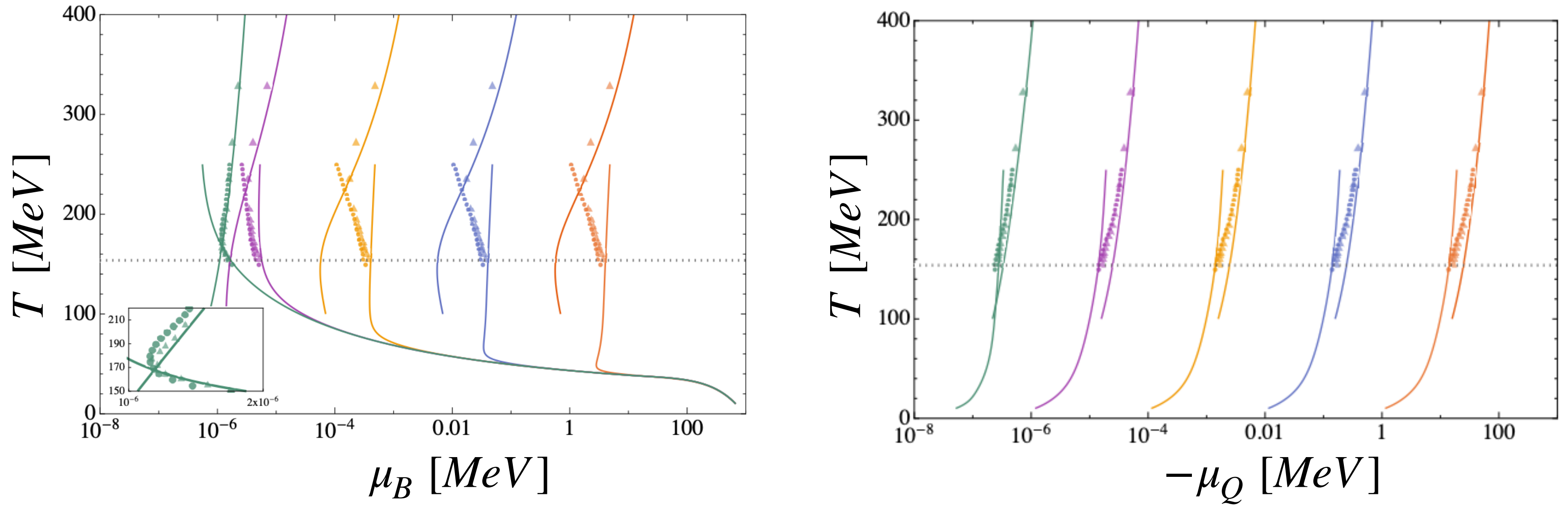
Phase Transitions and Critical point

- QCD phase structure and phase boundary shift differently in μ_B , μ_Q and μ_S (**BES Program**)



Cosmic trajectories

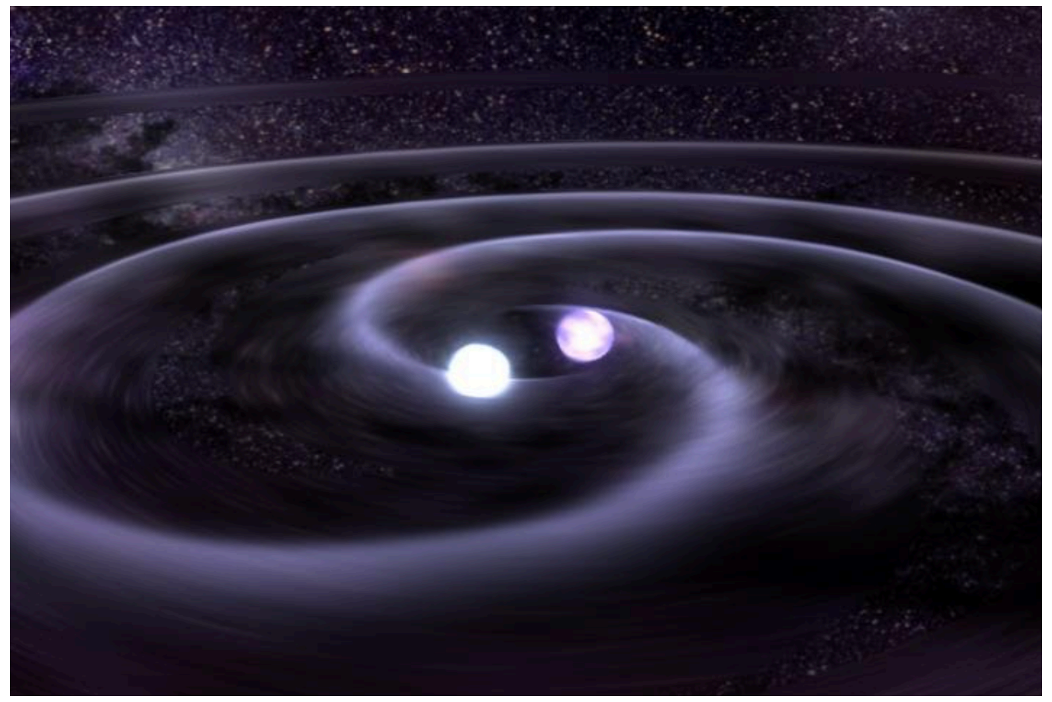
- μ_B, μ_Q and μ_S are important for cosmic trajectories



[Mandy M et al PRL. 121 20, 201302(2018)]

Neutron Star mergers

- To consider Asymmetric matter content



"The 4D EoS is in high demand for describing strongly interacting matter dynamics".

Taylor-expanded Lattice QCD

2D Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!} \chi_{2n}(T,\mu_B=0) \left(\frac{\mu_B}{T}\right)^{2n}$$

$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!} \left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\Big|_{\mu_B=0}$$

4D Taylor Expansion around $\hat{\mu}_B = \mu_B/T = 0$

$$\frac{P(T,\hat{\mu}_B,\hat{\mu}_S,\hat{\mu}_Q)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ}(T) \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

$$\chi_{ijk}^{BQS}(T) = \frac{\partial^{i+j+k}(P/T^4)}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\Bigg|_{\hat{\mu}_B,\hat{\mu}_Q\hat{\mu}_S=0}$$

[J. Noronha-Hostler et al Phys.Rev.C 100 6, 064910 (2019)]

[A Monnai et al Phys.Rev.C 100 2, 024907 (2019)]

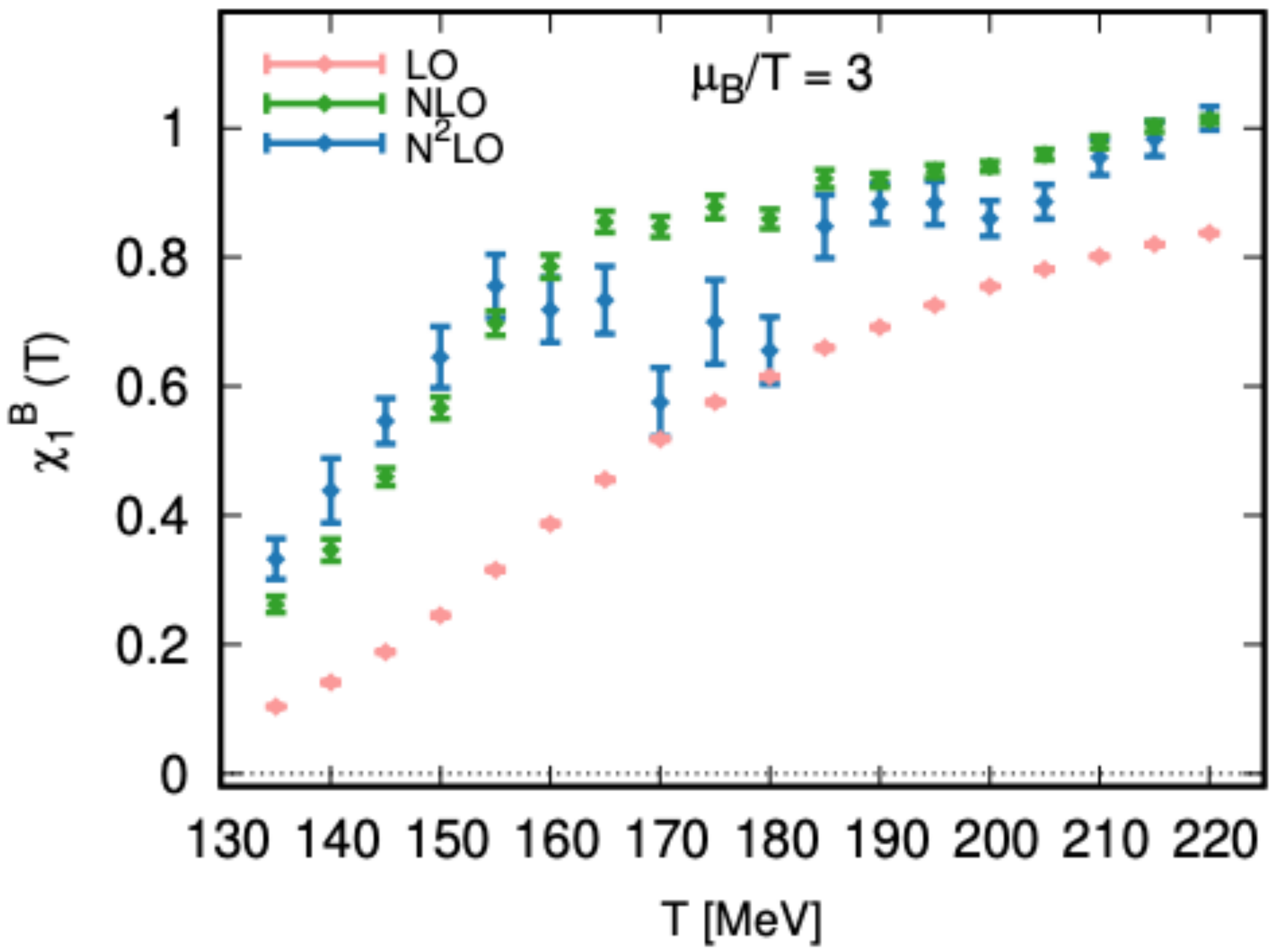
[A Monnai et al Phys.Rev.C 110 4, 044905 (2024)]

Limitations

- Currently limited to $\mu_i/T \sim 2.5 - 3,\ i = \{B,Q,S\}$ despite great computational effort.
- Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series.

[Bollweg, D. et al Phys.Rev.D 108 1, 014510 (2023)]

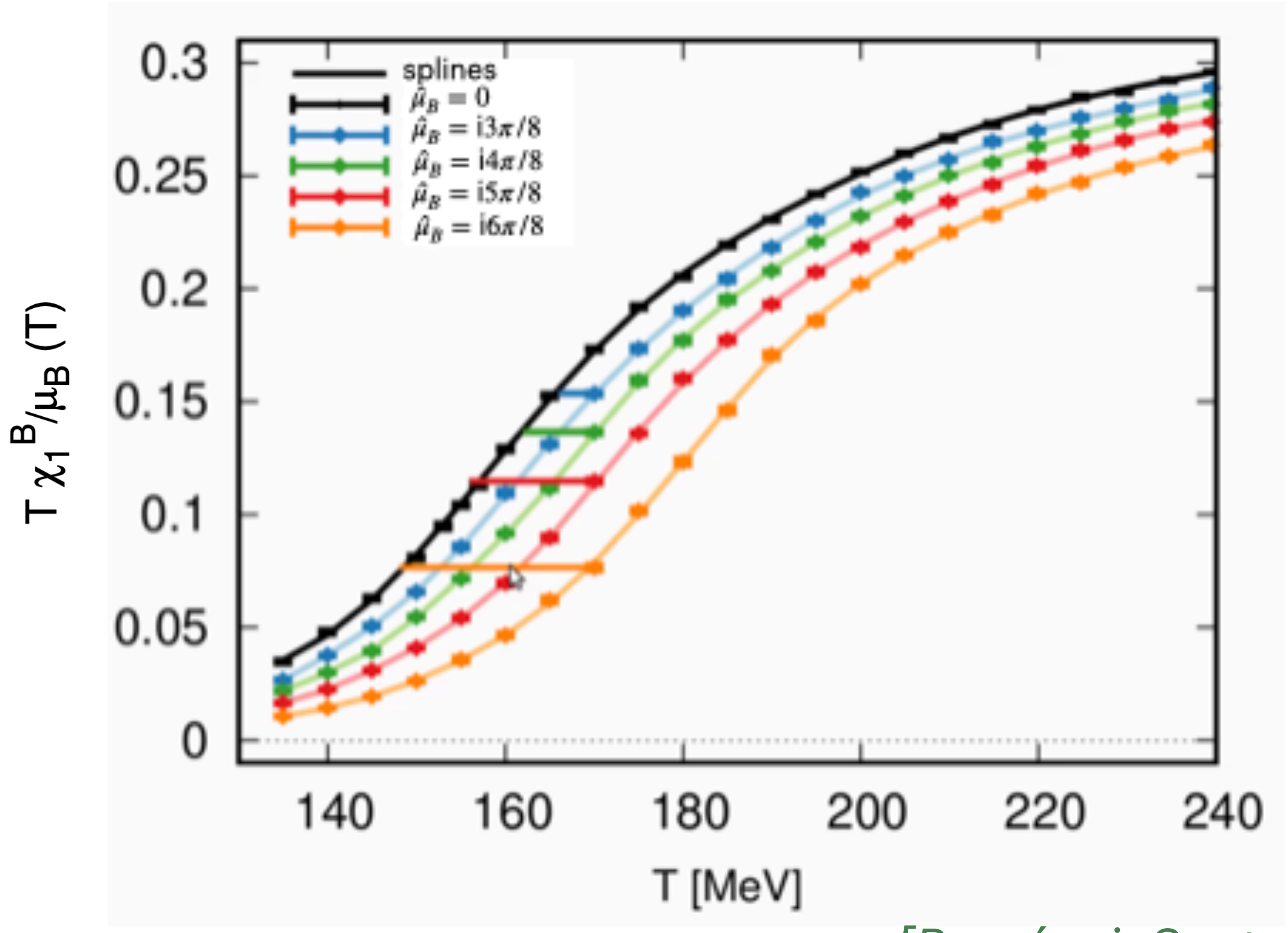
[Borsanyi , S et al Phys.Rev.D 110 1, L011501 (2024)]

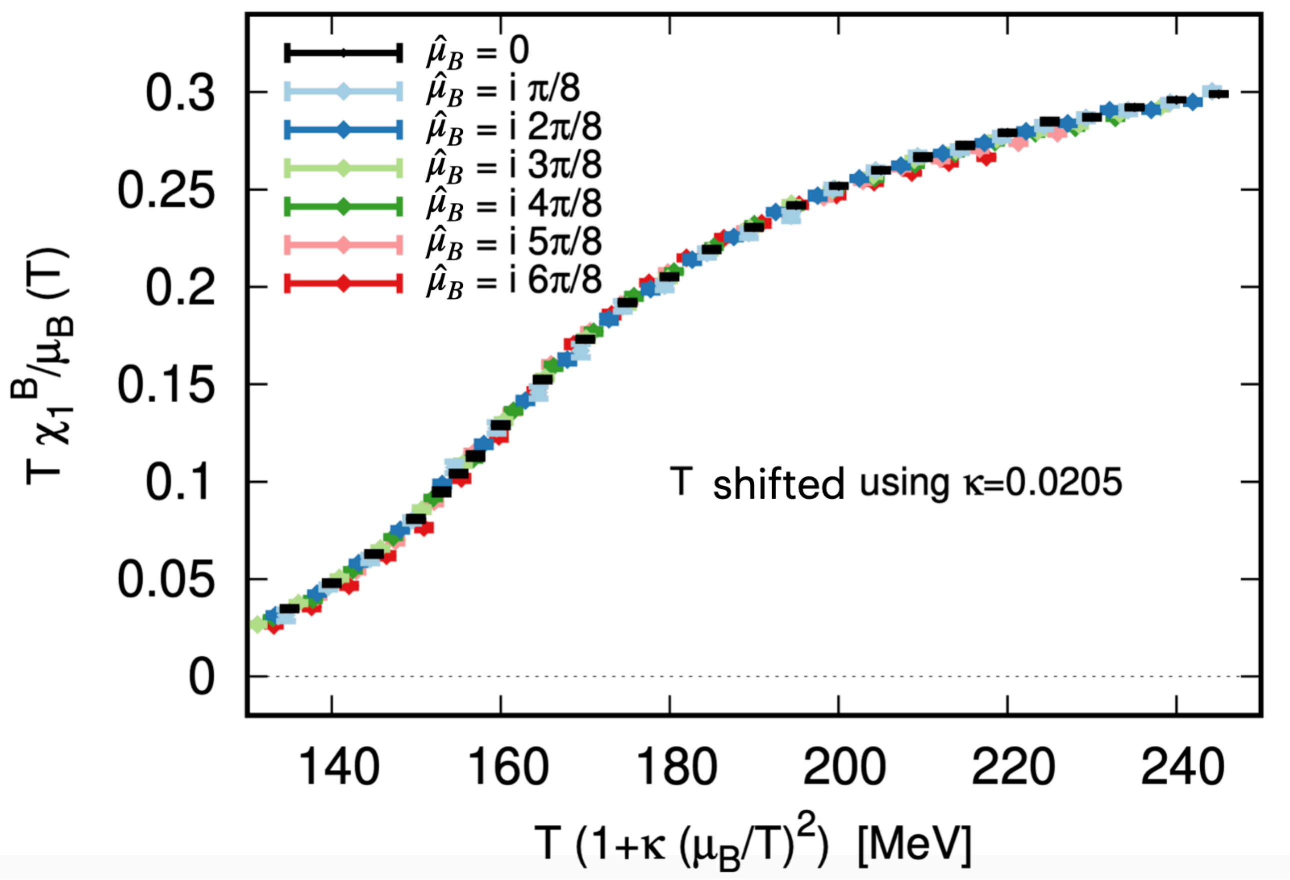


[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

T' - Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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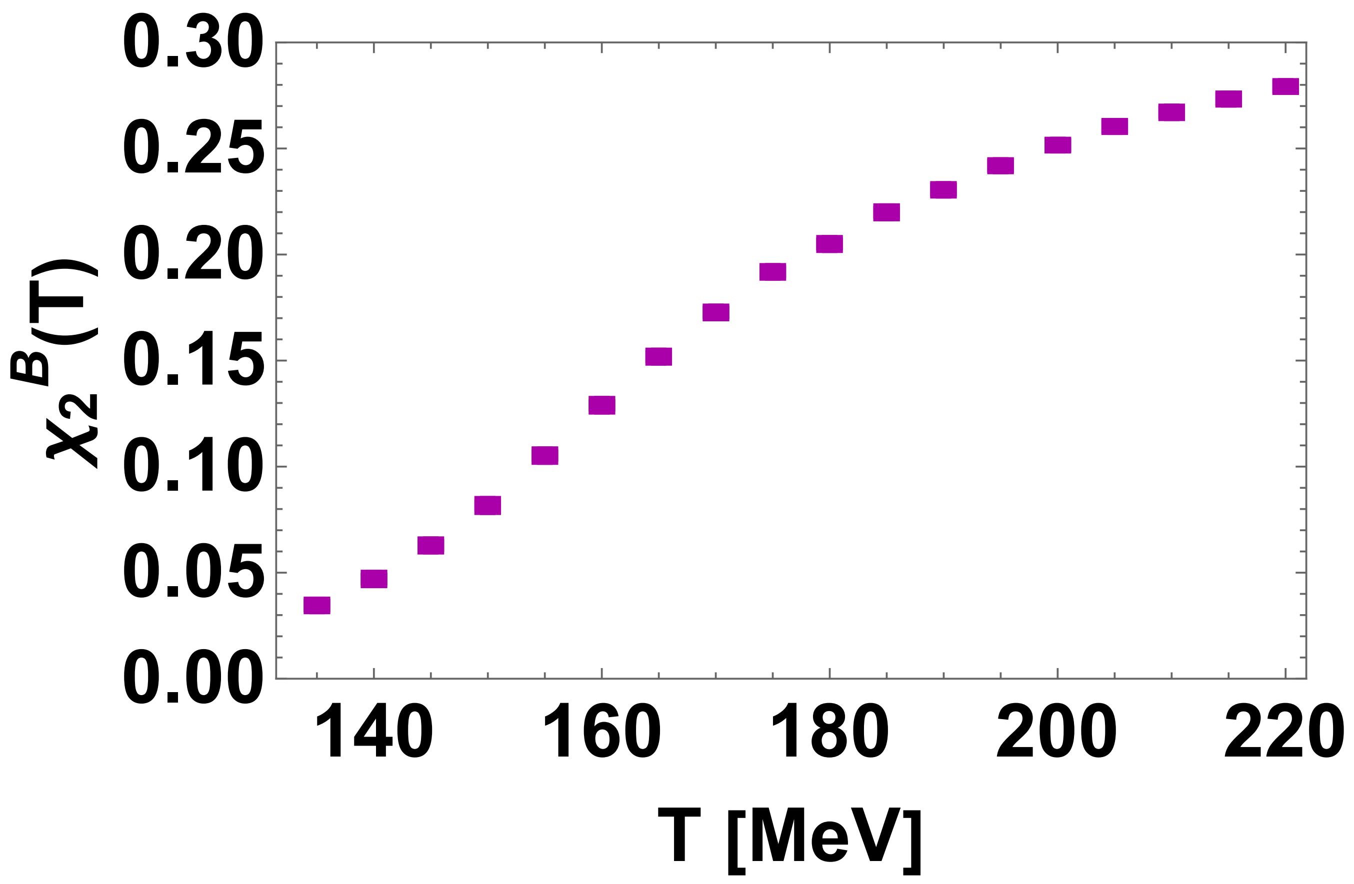


[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

Pros

- Uses few expansion terms.
- μ_B dependence is captured in T'-rescaling.
- Trusted up to μ_B/T=3.5 in the region where Critical point is expected.

[M K et al arXiv:2408.04588[nucl-th] (2024)]

T'-TExS with corrected SB-limit

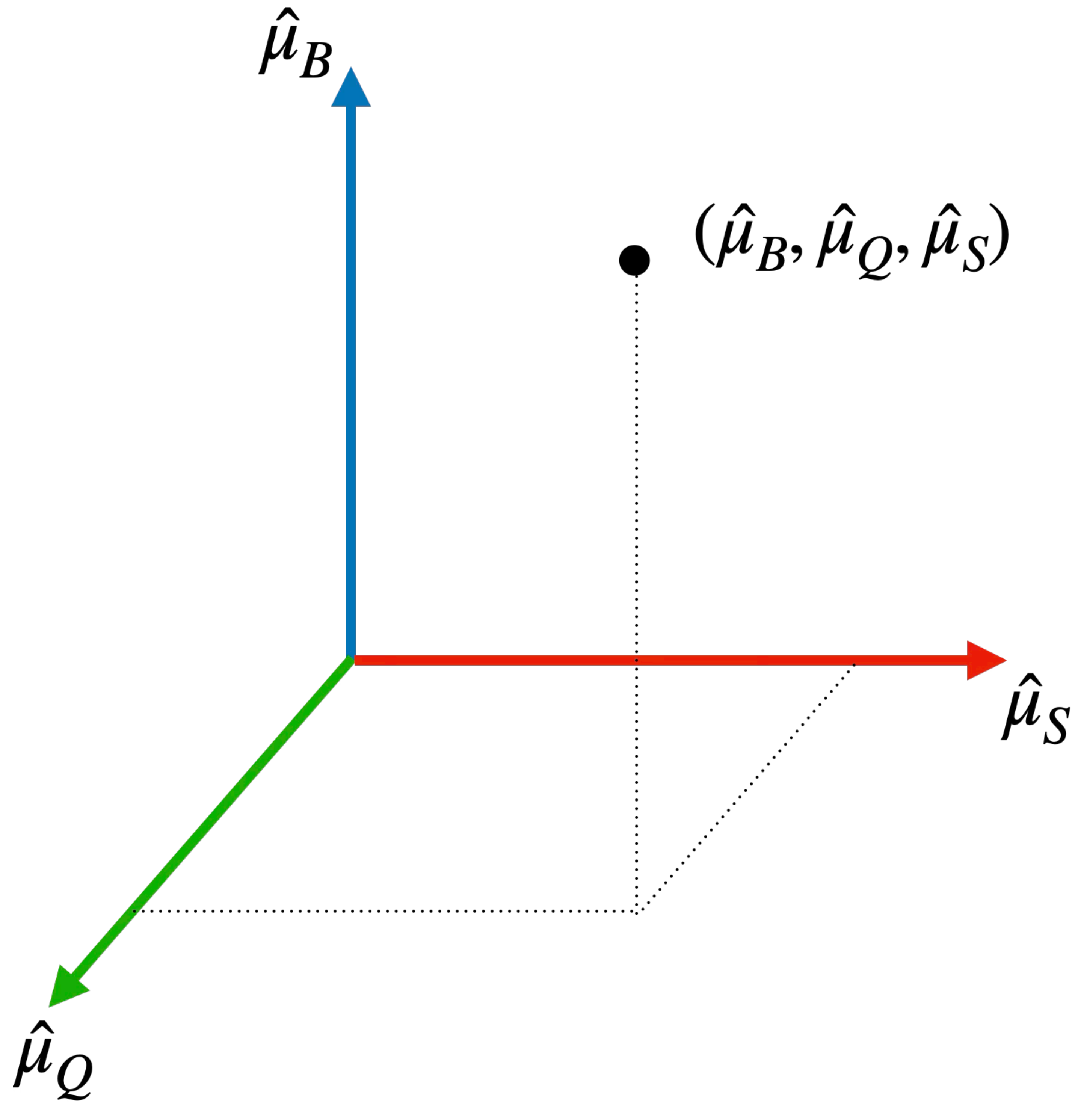
$$\frac{\chi_1(T,\hat{\mu}_B)}{\chi_1^{SB}(\hat{\mu}_B)} = \frac{\chi_2(T',0)}{\chi_2^{SB}(0)}$$

$$T'(T,\mu_B) = T\left[1 + \lambda_2(T)\hat{\mu}^2 + ..\right]$$

$$\lambda_2(T \to \infty) = 0$$

[Borsányi, S et al Phys. Rev. D 105, 114504 (2022)]

Generalization to 4D-T' expansion scheme

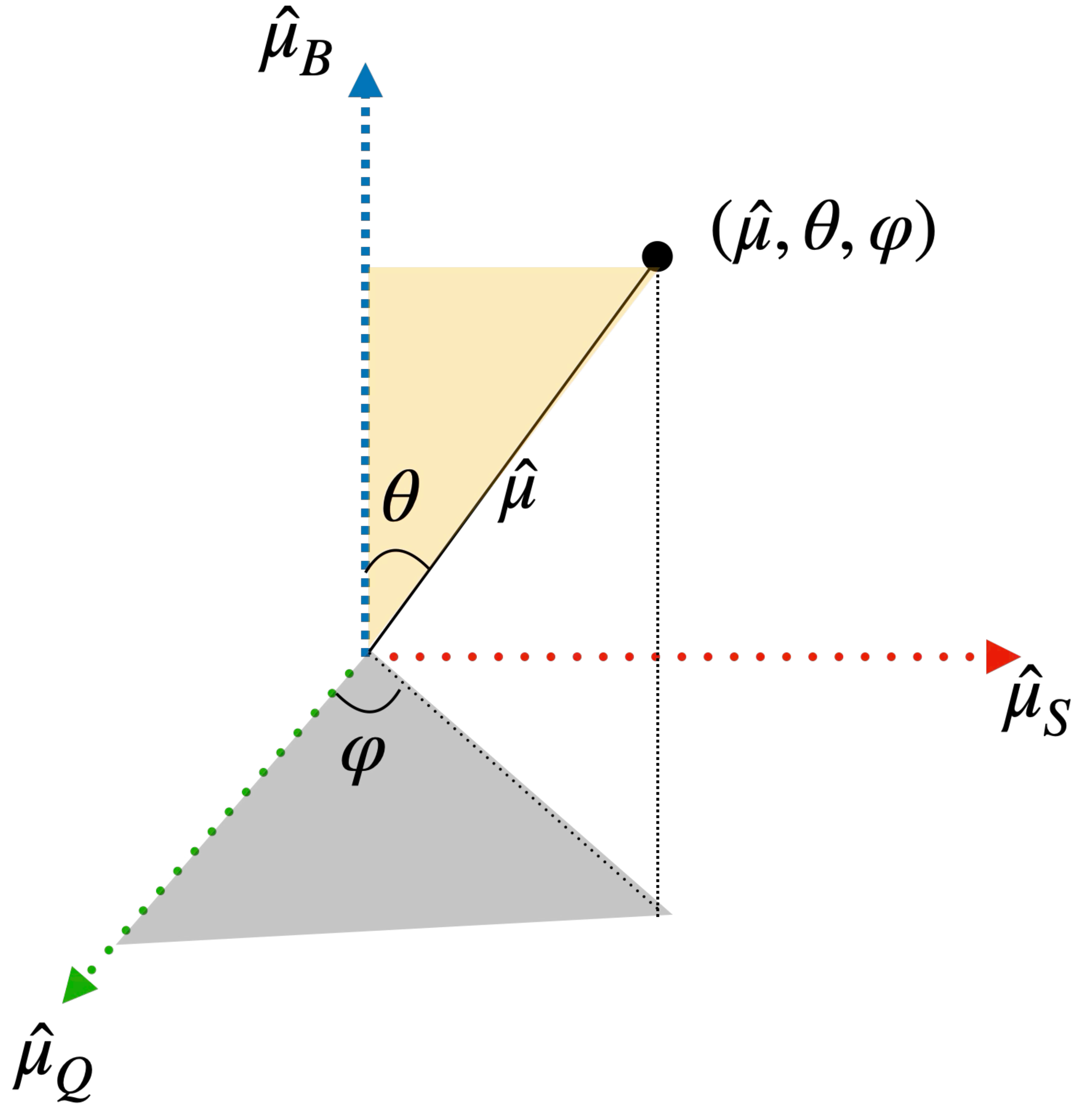


Cartesian coordinates

$T =$ constant

$$\hat{\mu}_B = \hat{\mu} c_\theta$$
$$\hat{\mu}_Q = \hat{\mu} s_\theta c_\varphi$$
$$\hat{\mu}_S = \hat{\mu} s_\theta s_\varphi$$

$$\hat{\mu} = \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2}$$
$$\varphi = \arccos \frac{\hat{\mu}_Q}{\sqrt{\hat{\mu}_Q^2 + \hat{\mu}_S^2}}$$
$$\theta = \arccos \frac{\hat{\mu}_B}{\hat{\mu}}$$



Spherical coordinates

2D T' Expansion

$$\frac{\chi_1(T, \hat{\mu}_B)}{\chi_1^{SB}(\hat{\mu}_B)} = \frac{\chi_2(T', 0)}{\chi_2^{SB}(0)} \longrightarrow$$

4D T' Expansion

$$\frac{X_1^{\theta,\varphi}(T, \hat{\mu})}{\overline{X}_1^{\theta,\varphi}(\hat{\mu})} = \frac{X_2^{\theta,\varphi}(T'^{\theta,\varphi}(T, \hat{\mu}), 0)}{\overline{X}_2^{\theta,\varphi}(0)}$$

$$T'^{\theta,\varphi}(T, \hat{\mu}) = T\left(1 + \lambda_2^{\theta,\varphi}(T)\hat{\mu}^2 + \ldots\right)$$

Apply 2D T′-expansion a long a given trajectory at fixed θ and φ.

Generalization to 4D-T' expansion scheme

Generalized Susceptibilities

$$X_n^{\theta,\varphi}(T) = \left.\frac{\partial^n p(T,\hat{\mu},\theta,\varphi)/T^4}{\partial\hat{\mu}^n}\right|_{\hat{\mu}=0}^{\theta,\varphi}$$

- Linear combination of χ_{ijk}^{BQS} **(Lattice data + HRG)**

$$X_2^{\theta,\varphi}(T) = c_\theta^2\chi_2^B(T) + s_\theta^2 c_\varphi^2\chi_2^Q(T) + s_\theta^2 s_\varphi^2\chi_2^S(T) + \ldots$$

$$X_4^{\theta,\varphi}(T) = c_\theta^4\chi_4^B(T) + s_\theta^4 c_\varphi^4\chi_4^Q(T) + s_\theta^4 s_\varphi^4\chi_4^S(T) + \ldots$$

Expansion parameter λ_2

$$\lambda_2^{\theta,\varphi}(T) = \frac{1}{6T(X_2^{\theta,\varphi}(T))'}\left(X_4^{\theta,\varphi}(T) - \frac{\overline{X}_4^{\theta,\varphi}(0)}{\overline{X}_2^{\theta,\varphi}(0)}X_2^{\theta,\varphi}(T)\right)$$

Generalized Charge density

$$X_1^{\theta,\varphi}(T,\hat{\mu}) = \frac{\overline{X}_1^{\theta,\varphi}(\hat{\mu})}{\overline{X}_2^{\theta,\varphi}(0)}X_2^{\theta,\varphi}(T'^{\theta,\varphi}(T,\hat{\mu}),0)$$

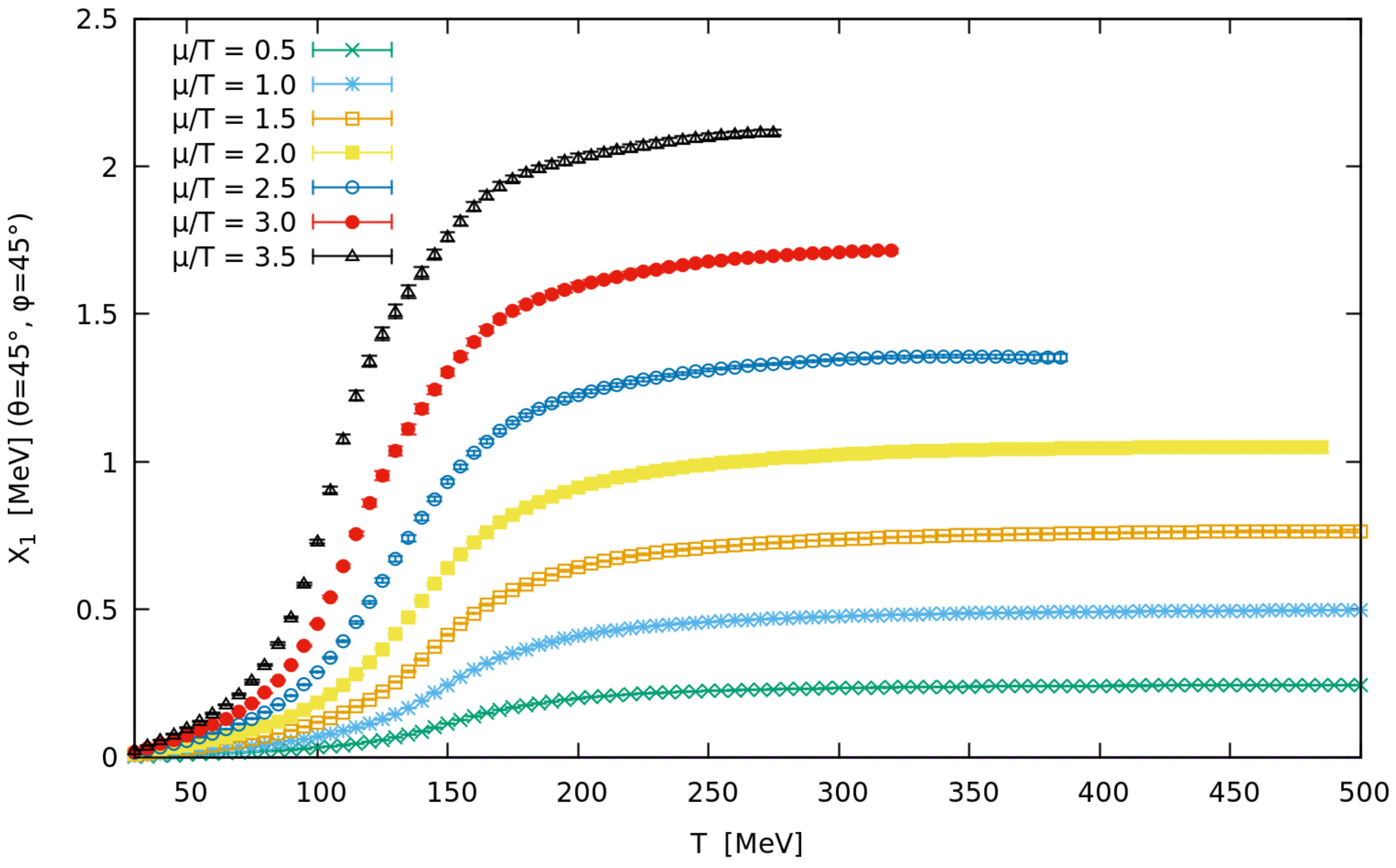
Example:

For $(\theta = 45^0, \varphi = 45^0)$

4D T' Expansion

$$\frac{X_1^{\theta,\varphi}(T,\hat{\mu})}{\overline{X}_1^{\theta,\varphi}(\hat{\mu})} = \frac{X_2^{\theta,\varphi}(T'^{\theta,\varphi}(T,\hat{\mu}),0)}{\overline{X}_2^{\theta,\varphi}(0)}$$

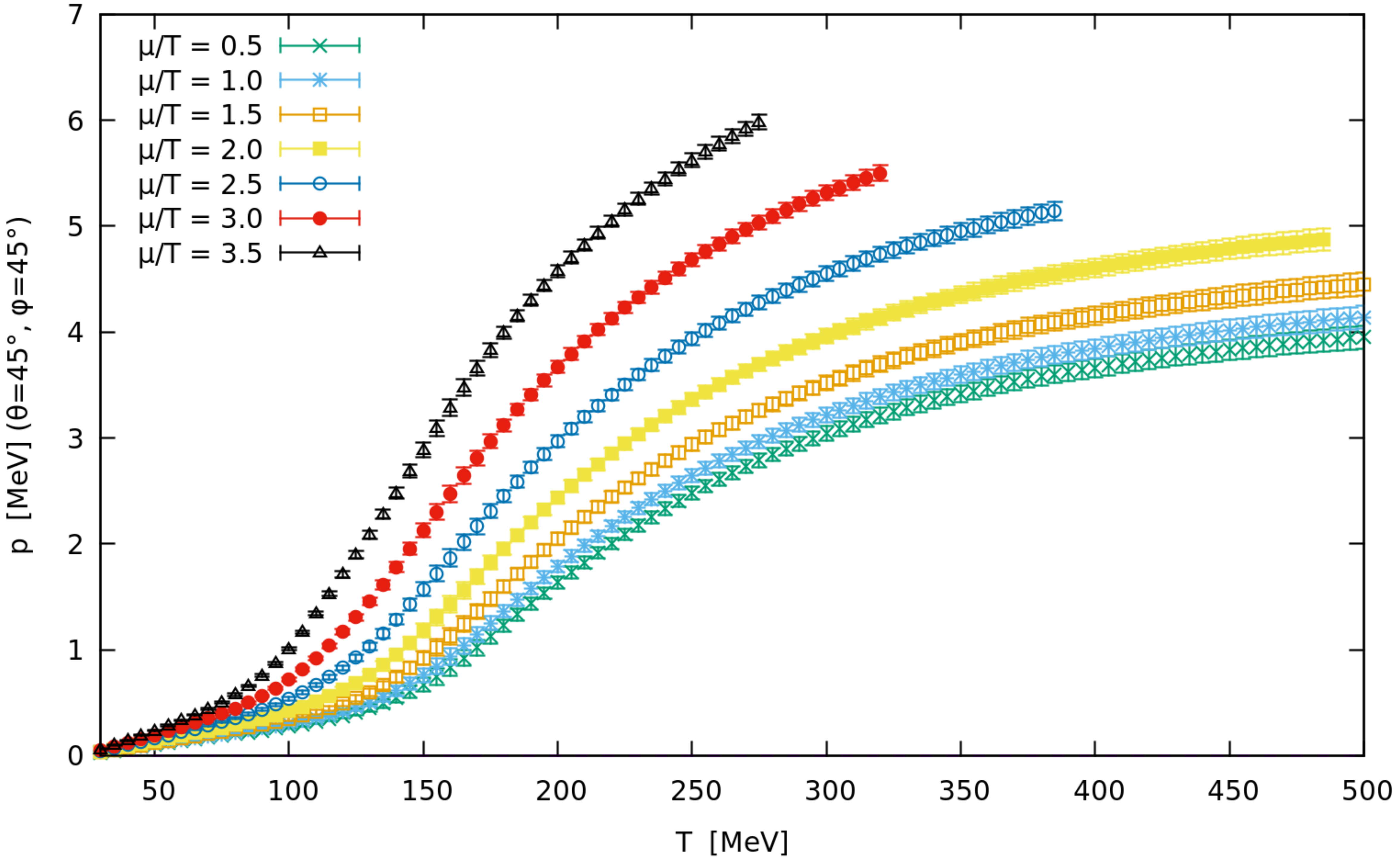
$$T'^{\theta,\varphi}(T,\hat{\mu}) = T\left(1 + \lambda_2^{\theta,\varphi}(T)\hat{\mu}^2 + \ldots\right)$$



Thermodynamic Observable

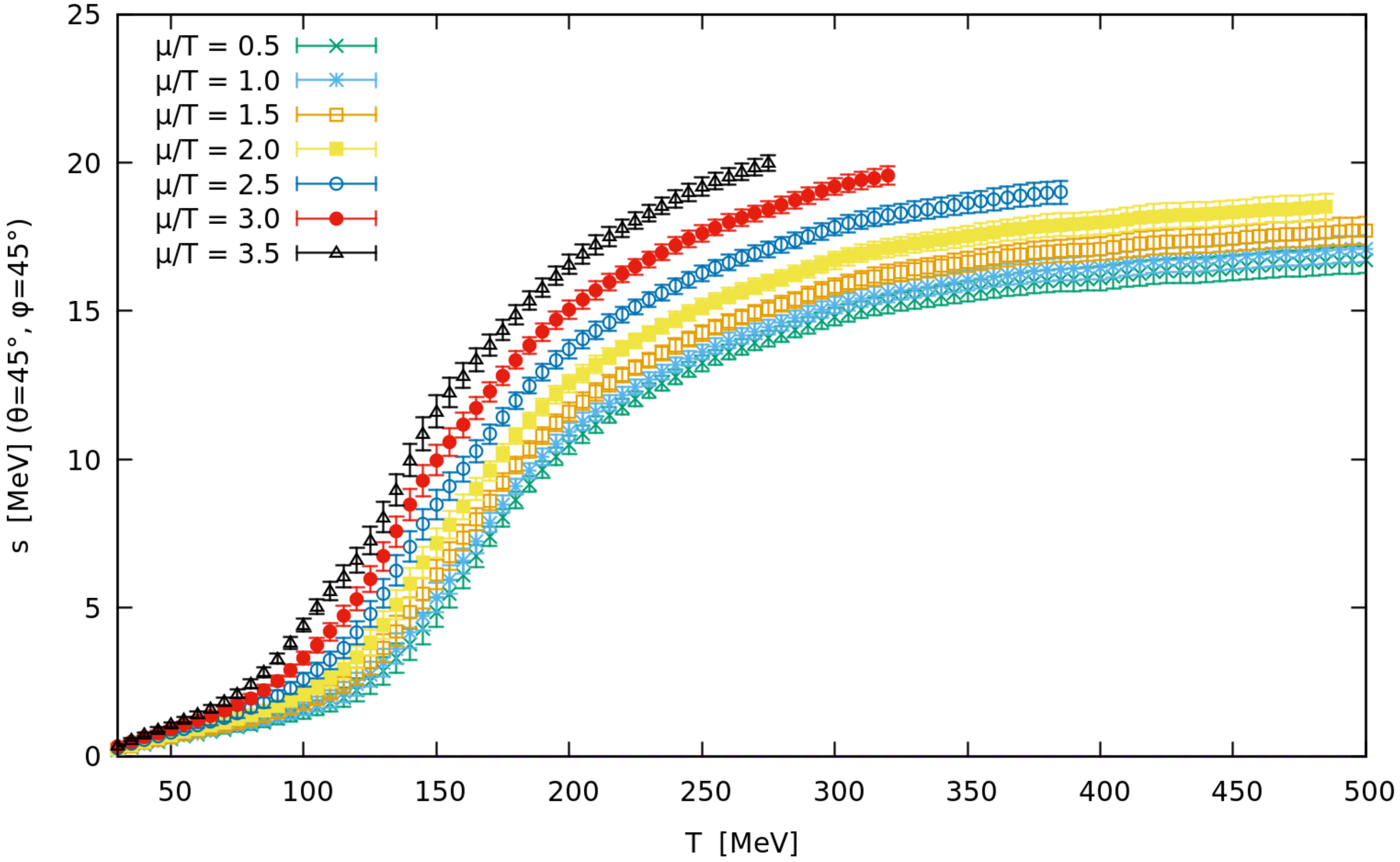
Pressure

$$\hat{p}(T,\hat{\mu}) = \hat{p}(T,\hat{\mu}=0) + \int_0^{\hat{\mu}} d\hat{\mu}' X_1(T,\hat{\mu}')$$



Entropy

$$\hat{s}(T,\hat{\mu}) = \left.\frac{\partial \hat{p}(T,\hat{\mu})}{\partial \hat{T}}\right|_{\mu}$$



Conclusion and Outlook

- We have generalized the Lattice T'-Expansion Scheme to give 4D $(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)$ EoS.
- This construction includes a comprehensive and well-propagated **error analysis** based on new lattice data.
- We have constructed a **4D lattice EoS (BQS-TExS)** that show no unphysical behavior at **higher densities**.

Outlook

- Implement 4D-TExS in MUSES Calculation Engine

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Thank you!